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Fundamentals of Submarine Concept Design

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ABSTRACT

Many papers have been presented on the subject of submarines, but there has been very little on the basic design process. Most concept design is done within the confines of the Naval establishments and therefore not made public. A concept design course has been taught at the Massachusetts Institute of Technology. The concepts presented here have been developed over a number of years. They have been tested by the many students who have taken the course. The paper covers the fundamentals of all phases of the process. However due to the limitations of space, discussion of each phase is very brief. A system for the collection of data for future use can be enhanced by use of the curves presented. Also, computer programs can be developed by the use of spread sheets that will enhance the many trade off studies that are required for a successful concept design.

1. INTRODUCTION

A landmark paper on submarine design was presented by Arentzen and Mandel (1961). Since that time, some major developments have taken place that impact on the ability to conduct feasibility studies of submarines. One is the increase in the use of nuclear power which has an almost unlimited power source resulting in the ability to maintain high speeds for long periods with essentially constant weight. The second is the resurgence of the body of revolution hull form, and the third is the explosion of the capabilities of the personal computers.

As pointed out in the earlier paper, the volume and the shape of the submarine hull are most important in all phases of the concept design. Any shape to be considered is subject to very rigorous examination by the use of calculations to any degree of sophistication desired. The simple form of a body of revolution can readily be described by simple geometric forms which in turn can be developed from elementary mathematical equations. The modern computer enables one to create a multitude of hull forms easily and quickly. The development of this philosophy occurred over the last 20 years of teaching MIT students the art of submarine concept design. Initially the concept was used only to check the work of the students and was reported in Jackson (1983). Eventually the concept was greatly expanded and has resulted in a very useful tool for investigating various concepts for analysis and trade off studies. This paper only describes the end product and does not discuss the many tangents that were tried and then later discarded or improved upon.

Figure 1 is a flow diagram for concept design. Different orders can be specified, but this one is quite satisfactory. Each of the categories will be discussed later in the paper. This order works only because equipment, storage spaces, and crew accommodations have been developed to have a weight density of about one. The symbols used in this paper are those set forth in Comstock (1967), pp. 461, 604-606, 717.

2. BACKGROUND

The naval architecture of submarines is exactly the same as for surface ships, the only difference being that submarines have to operate in two basic conditions, one on the surface and the other completely submerged. The basic laws of naval architecture are the same for both conditions. Best operation on the surface for any high-speed vessel requires a long thin body with a very sharp entrance at the water line. A completely submerged vessel, operating some three diameters or more below the surface, can have a rather blunt nose and a fine run, sometimes called the after body. Many fluid mechanics text books, *e.g.*, Dodge and Thompson (1937), indicate that the optimum length/diameter ratio is in the range of four to six. This optimum has not been reached by any major submarine since the HOLLAND type at the beginning of this century. As more and more equipment is required in present submarines, it is unlikely that this optimum will ever be obtained due to the limitations on draft in most of the harbors of the world. The advent of stronger steels and better computational abilities has enabled operations at deeper depths. Perhaps the most gains have been made in the areas of hydrodynamics. This is again due to the increased capability of the computers. The

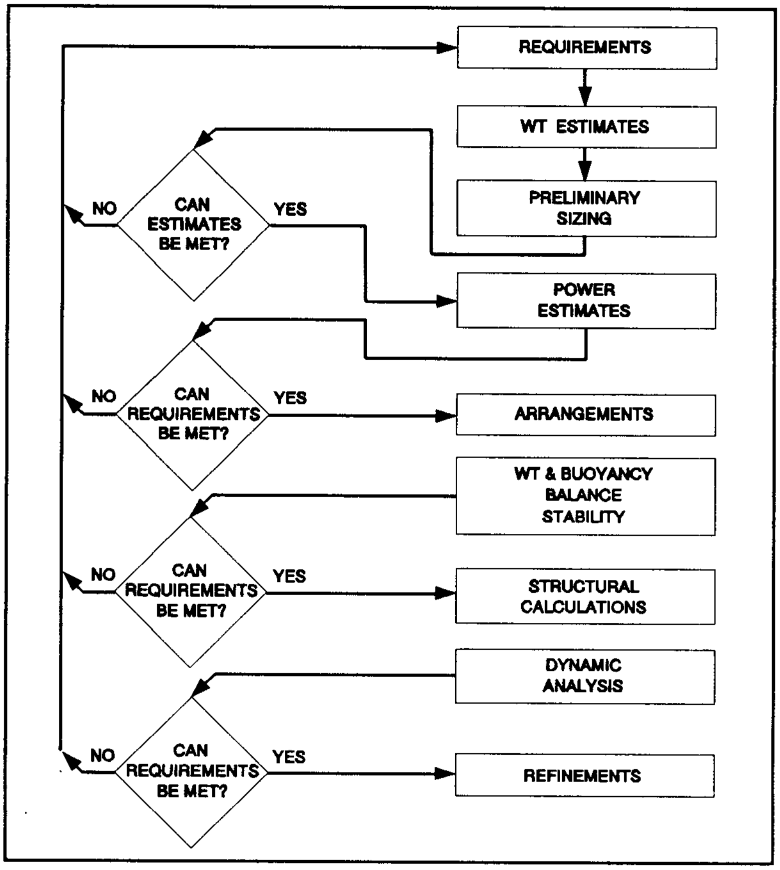


Figure 1. Feasibility-Study Flow Chart

concepts outlined in this paper have been developed by the author, but credit must be given to the many students in the MIT professional summer program.

3. WEIGHT AND VOLUME

Archimedes provided the basic fundamental of all naval architecture by stating that a floating vessel will displace a volume of fluid whose weight exactly equals the weight of the vessel. As in all ships the weight and the underwater volume are explicitly tied together. Another and perhaps more important need for volume is to provide space for all of the equipment and personnel required to operate and maintain it. In order to start the concept design phase, it is necessary to have a basic idea of the final product. For example: "Will it be missile carrying or an attack type?" A set of requirements are mandatory prior to any design work. In a successful design, they are mission driven. These are usually provided by the customer. If they are not, it will be necessary for the designer to develop them.

The essence of the concept design is the weight estimating, as everything else is subject to a rather exact computational analysis. It is imperative that one have a data base that has been developed over a considerable number of successful designs. Daniel (1983) has given an outline for the accumulation of weight data. This system had its origins in the days of the sailing Navies, which accounts for the breakdown. Any system will work, but it must be consistent and be in use for many years before it will be practical. The Washington Treaty of 1922 reinforced the above plan. It is the one that will be used throughout this paper.

There are several kinds of displacements in the submarine language, and one has to be careful to define which one is being referred to in the discussion. The normal surface condition (NSC) is easily defined as being the sum of the fixed weights plus the weight of the fixed ballast and the variable load. The sum of the fixed weights is known as condition A-1. When the fixed ballast is added to the A-1 weights, it is known as the condition-A weights. At the Washington Treaty, this was known as the standard displacement. It is also sometimes referred to as the light ship weight. The variable load is the sum of all of the items that can change from day to day plus the variable ballast required for the submarine to remain in equilibrium. A submarine, like all ships, can be caused to sink by adding weight to it. In order to accomplish this, large tanks (MBT, or main ballast tanks) are built into the hull. Figure 2 shows a possible location of the tanks. They are sized and located in positions that will enable the submarine to be in equilibrium both on the surface when the tanks are empty and submerged when they are completely full.

There are large spaces in the submarine hull which are difficult to make watertight. The solution to this

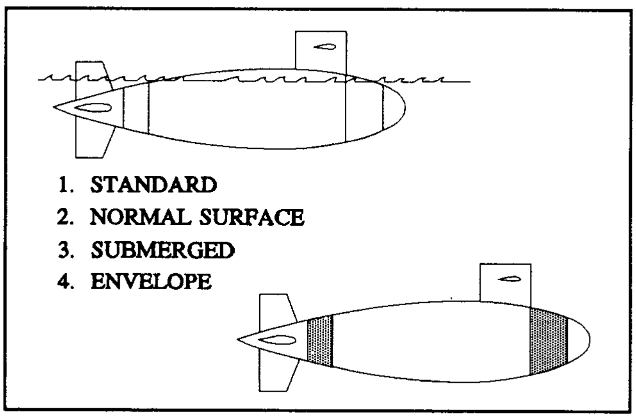


Figure 2. Kinds of Displacement

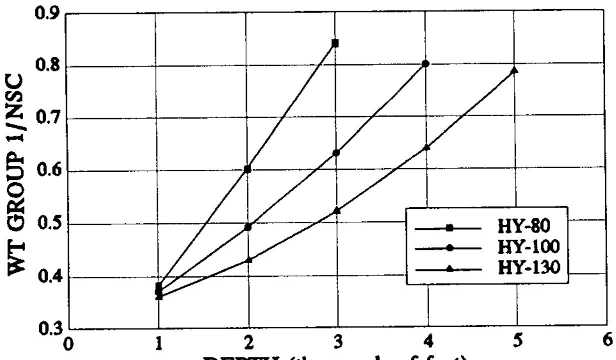


Figure 3. Weight Group 1 / NSC *vs* Depth

problem is to leave them open to the sea through small openings. They are then called free flood (FF) spaces. If the weight of the water in the MBT is added to the NSC the submarine will sink and the result is known as the submerged displacement (SUBD). When the FF is added to the SUBD the result is known as the envelope displacement (ENVD). The displacements of the structure in the fairwater and appendages are a part of the envelope displacement. However, since these are small compared to the hull, they can be ignored during the concept phase. With so many definitions of displacement, one can see the confusion that might exist if one is not careful to specify which one is being discussed. The ENVD will be referred to many times in this paper and it is the basis of much of the later developments. Using the above approach, Table 1 can be made, which will indicate the usefulness of the concept.

There are some weight ratios that are rather consistent for a given type of submarine. These appear to be quite independent of the national origin of the design. The ratio of weight of structure, GR1 (see Table 1), to NSC, *i.e.*, GR1/NSC, is relatively constant for a given material and operating depth. It usually does not vary significantly with size of the structure. Figure 3 indicates some ballpark ratios of GR1/NSC for various materials. The upper curve is for HY-80, the middle for HY-100, and the bottom for HY-130. Figure 4 contains curves relating the unit weights of machinery as a function of total shaft horsepower (SHP). The basic data was extracted from Powell (1958), p. 737. The two lower curves are the author's predictions. These curves can be used to estimate the weight of Groups 2 and 3 (see Table 1). Group 4 weights are a function of the weapons systems, which are usually specified in the requirements. Groups 5 and 6 are a function of NSC. Percentages can be developed from other similar ships or the weights of the other submarines can be used directly or appropriately massaged. Group 7 weights are a function of the number and type of weapons specified. This group contains only the fixed weights such as tubes and handling gear. The amount of LEAD to be included is generally listed as a fraction of the A-1 weights. It is a function of the unknowns in the design and the stability requirements. The ratio LEAD/A-1 for concept design is usually on the order of .1 to .125.

The variable load, VL, includes all of the variable items to be carried, as well as the change in buoyancy due to changes in water density. The MBTs are a function of the NSC by definition. The usual range of the weight ratio MBT/NSC is .1 to .15 for modern submarines. The volume of the MBT is sometimes referred to as the reserve buoyancy. The FF is listed as a function of the envelope displacement. The ratio FF/ENVD is about .04 to .05 for single hulls and about .07 to .09 for double hulls. Using the above discourse, it is possible to make a rather simple relationship of all of the nine weights and NSC displacement as follows:

$$NSC = \frac{[1+\%LEAD]*\Sigma[2\text{-}7]}{1-\%VL-[\%GR1*[1+\%LEAD]]} \qquad (1)$$

Here the % sign represents percent/100, that is, the absolute ratios mentioned above. This simple relationship is very useful to answer the "what if" effects of adding or removing weights and or volumes.

Providing sufficient but not excessive volume to encompass all of the equipment and operating spaces is an art as well as a science, which requires a great deal of experience. Many of the items to be included have a re-

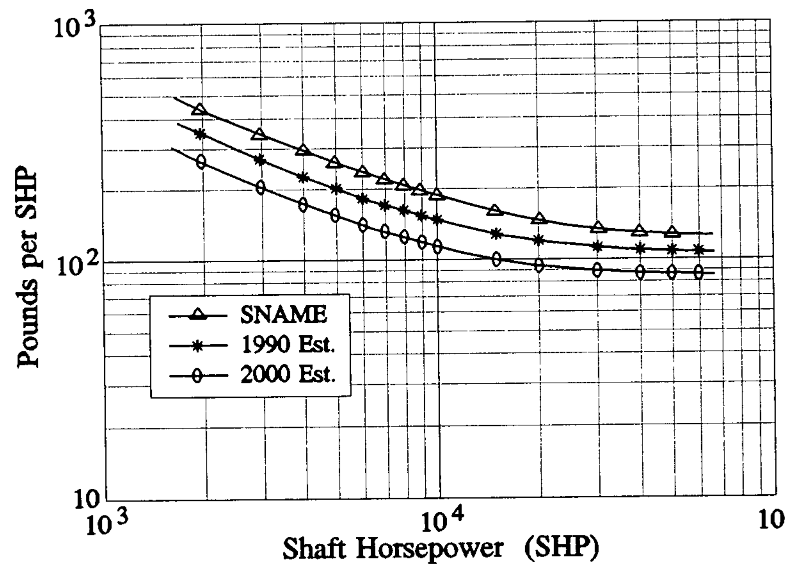


Figure 4. Machinery Weight Predictions

Table 1. Weight and Buoyancy Estimating

Group#	Name	Function of	Example (Long Tons)
1	Hull	NSC	2888.364
2	Mach	SHP & NSC	907
3	Elect	SHP & NSC	209.4
4	Electronics	Given	180.5
5	Aux Mach	NSC	579.8
6	Outfit	NSC	256.8
7	Weapons	Given	785.7
A-1	Σ(1-7)		5807.564
LEAD	A - A-1	Requirement	936.4698
A	Σ A-1 & LEAD		6744.034
VL	Var Load	NSC	581.2623
NSC	Σ VL & A	NSC	7325.296
MBT	MBT	Requirement	929.5801
SUBD	Σ NSC & MBT		8254.877
FF	Free Flood	ENVD	388.9732
ENVD	Σ SUBD & FF		8643.85

% Group 1	0.3943	C_{wsa}	0.750606
% LEAD	0.16125	C_{wsf}	0.893635
% VAR LOAD	0.07935	DRAFT FWD	31.050
% MBT	0.1269	DRAFT AFT	31.100
% FREE FL	0.045	SURFACE D	8243.042
		SURFACE LCB	153.011
n_f	3	TRIM ANGLE	0.008527
n_f	2.75	(+ is nose-up)	
DIAM	38	C_p	0.792839
K1	1.832	C_{ws}	0.86959
K2	1.153	*L/D*	8.842105
LOA	336	*L/D* - K1	7.01037
WS	34880.88	*L/D* - K2	7.689009

quirement for deck space. Others are given in volume and still others are given by bulk dimensions. Over the years, equipments have been developed such that the total weight density of the submarine is about equal to that of sea water. A data bank of weight and volume for all components is essential for good concept designs. Perusal of the previous weight summary will show that the volume of the pressure hull divided by 35 must equal the NSC displacement, since seawater occupies approximately 35 cubic feet per ton (corresponding to 64 pounds per cubic foot). There are examples of submarines that are weight or volume limited. It is desirable to have a submarine that has neither limitation but has margin for growth in both categories. Other limitations are stability and longitudinal center of gravity (LCG) of the lead.

There are discrete hull diameters that provide the best utilization of space. This comes about because there are two or more decks in a submarine. If the diameter is too large, the deck heights will be too high, which makes their utilization difficult. If the diameter is too small, the deck heights will not provide enough headroom.

The principles involved in the volume calculations are the same for any hull form. The current selections of the bodies of revolution make the problem much simpler, since all of the sections are circular, by definition. The usual mathematical methods can be used for calculations. Simpson's Multipliers is the one recommended.

Froude (1877) introduced the concept of a ship with a forward end called the entrance, a parallel middle body, and an after end called the run. Chapman (1768) introduced the concept of a ship hull with the entrance a portion of a parabola of revolution, and with the run a portion of an ellipsoid of revolution. He towed models of this arrangement, but at too high a speed, so that the concept was delayed nearly 100 years. This concept is tailor-made for use in calculating volumes of modern submarine hulls, as described by Jackson (1983). It was developed by assuming a body of revolution with a length/diameter (*L/D*) ratio of six and a maximum diameter at .4*L*. The entrance has a length, L_f, of 2.4 diameters. The run or after end has a length, L_a, of 3.6 diameters. The entrance can be calculated as an ellipsoid of revolution, and the run as a paraboloid of revolution which is rotated about a line parallel to the directrix. The equations of the offsets for each are given below. The hull radius at each station can be found by multiplying the offsets by half the maximum diameter, *D*/2.

If one were to use equations for true ellipsoids and parabolas, the entrance and the run would be too fine for a modern submarine. The displacement can be increased by using larger exponents (n_f and n_a), as in Equations 2 and 3. If even more displacement is required, a parallel middle body of cylindrical shape can be inserted at the maximum diameter. The prismatic coefficient, C_p, for a cylinder is 1. Using the above concept, the length of the parallel middle body (PMB) is the length overall less 6*D*, that is, LOA - 6*D*.

$$y_f = \frac{D}{2}\left[1 - \left(\frac{x_f}{L_f}\right)^{n_f}\right]^{1/n_f} \qquad (2)$$

$$y_a = \frac{D}{2}\left[1 - \left(\frac{x_a}{L_a}\right)^{n_a}\right] \tag{3}$$

Here x_f and x_a are the distances from the maximum diameter. The Appendix contains a compilation of the nondimensional offsets, C_p, C_{ws} (wetted-surface coefficient), and LCB/L (LCB = longitudinal center of buoyancy), as functions of the exponents n_f and n_a; Figures 22 and 23 show nondimensional plots of entrance and run offsets, respectively. With these concepts, a very simple method of calculating the volume of the entire hull can be developed. This is true for the ends separately and for the PMB. Let V_f, V_a, and V_{PMB} denote, respectively, volume of the entrance, the run, and the parallel middle body, and let C_f and C_a be the prismatic coefficients. The resulting equations are:

$$V_f = \pi(D/2)^2[C_{pf} * 2.4 * D]$$

$$V_a = \pi(D/2)^2[C_{pa} * 3.6 * D]$$

$$V_{PMB} = \pi(D/2)^2[L - 6D]$$

$$\textit{Total Volume} = V_f + V_a + V_{PMB}$$

The above can be combined into the following:

$$V = \frac{\pi D^3}{4}\left[3.6\,C_{pa} + L/D - 6 + 2.4\,C_{pf}\right]$$

Figure 5 illustrates this concept. If C_{pa} and C_{pf} are selected as functions of the exponents n_a and n_f, the value of the terms in the brackets can be calculated and tabulated as L/D - K1, where

$$\mathrm{K1} = 6 - 2.4\,C_{pf} - 3.6\,C_{pa}$$

In seawater at 35 ft^3/ton, the envelope displacement is (in long tons):

$$\mathrm{ENVD} = \frac{\pi D^3}{4 * 35}\left[L/D - \mathrm{K1}\right] \tag{4}$$

Using the same rationale, a similar equation can be developed for calculating the wetted surface, WS, which reduces to:

$$\mathrm{WS} = \pi D^2\left[L/D - \mathrm{K2}\right] \tag{5}$$

where

$$\mathrm{K2} = 6 - 2.4\,C_{wsf} - 3.6\,C_{wsa}$$

The wetted surface area is required to estimate the shell weights and to calculate the speed-power relationship. Figures 6 and 7 contain curves of K1 and K2 as functions of n_a and n_f. Study of the above will show that L_a and L_f could be varied instead of n_a and n_f. Whichever method is used, the resulting hull form will be essentially the same. The calculation of the hull form for a body of revolution has been reduced to L, D, two exponents, and adjustable constants. Hull forms from Series 58, mentioned in Arentzen and Mandel (1960), can be matched very closely. Returning to Equation 4 and using the data from the weight summary, Table 1, one can select a diameter and solve the equation for L. The principle dimensions are now a basis for proceeding with the design.

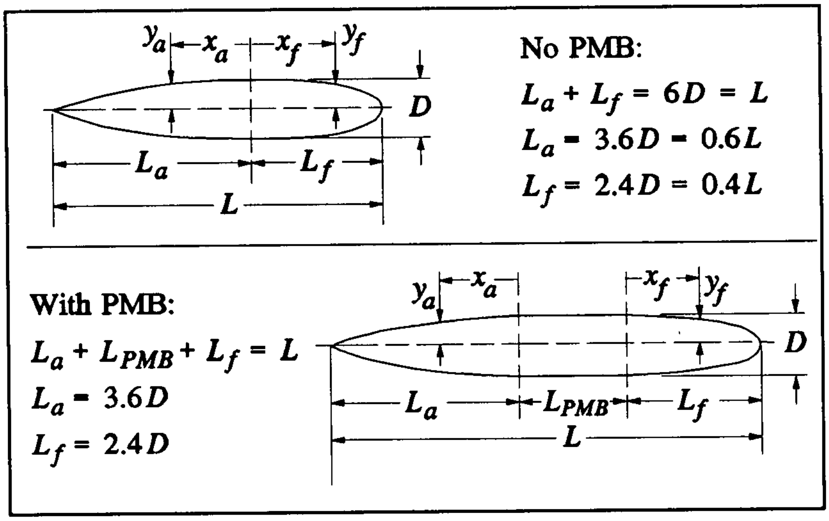


Figure 5. Geometry of a Submarine

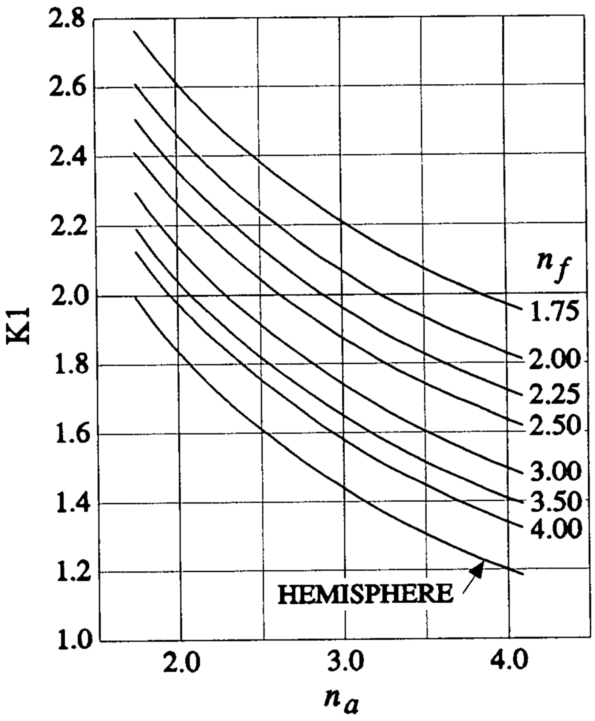


Figure 6. K1 as a Function of n_a and n_f

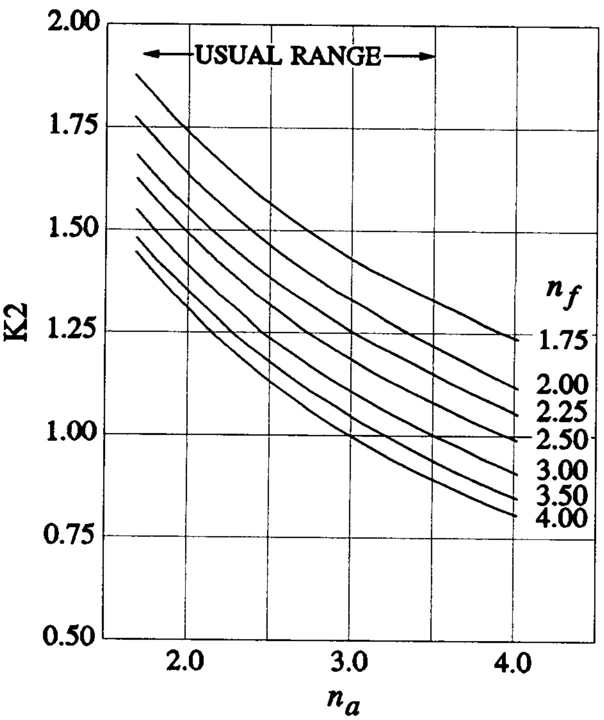


Figure 7. K2 as a Function of n_a and n_f

The location of the weights is as important as the weights themselves. The best way to locate the weight groups is to make a small sketch to scale. It does not have to be very pretty. A free-hand one will be quite satisfactory. With this sketch, both the horizontal and vertical centers can be estimated and Table 2 created. The group weights and their arms are straightforward. When they are entered, it is a simple matter to determine the moments by multiplication and then sum them up. Since the weights must equal the buoyancy, the lower end of the table can be volumes divided by 35 ft^3/ton.

Determining the volumes is just a matter of using the equations of solid geometry. Offsets can be determined from Figures 21 and 22 in the Appendix. With them, the ENVD volume and buoyancy, the LCB, and the VCB (vertical center of buoyancy) can be determined by the use of Simpson's multipliers. They have to be calculated at the NSC draft as well as submerged. The buoyancy and arms of the FF can be estimated and subtracted from the ENVD to determine the SUBD. Using Figure 8, the LCB of the SUBD can be determined straightforwardly. The VCB of the ENVD will be at the axis of the body of revolution. There will usually be a requirement for BG (height of LCB above LCG) on the order of 1 to 1.25 feet in the submerged condition. If this is subtracted from the VCB, the required VCG will be determined. Now working towards the weights above NSC in Table 2, all of the entries can be considered as weight. The transition from buoyancy to weight has been made.

The next step is to find the draft of NSC and calculate the LCB and VCB. The draft, LCB, and VCB must be corrected for the free flood that is below the waterline while on the surface. The weights and moments of the NSC can be subtracted from the SUBD. The difference will be the MBT. The LCG of the MBT will determine the location of the tanks. If they are the correct size with their LCG at the calculated point, the submarine will be in equilibrium and level both on the surface and submerged. If the MBT LCG is not at the calculated point, the submarine will have a trim on the surface if it has none while submerged. Subtracting the variable load from NSC results in Condition A. The difference between A and A-1 is the amount of lead carried. The LCG and VCG can be determined from the moments. If the lead is not somewhere near the LCB submerged, it will not be satisfactory and adjustments will have to be made.

The lead is generally divided into stability and margin categories. An equation of weights and two of moments can be set up. There are six unknowns, but three can be arbitrarily specified. They are the LCG and VCG (vertical center of gravity) of the margin lead and the VCG of the stability lead. The solution of the three remaining equations will determine the amount of stability and margin lead and their LCG and VCG. Frequently too much lead is required for stability and not enough for margin. It is then necessary to review the VCG estimates of the weights and lower them where possible. One must remember that, at this stage of the design, budgets for weight and centers are being established for the detail designers that are to come into the design process later on.

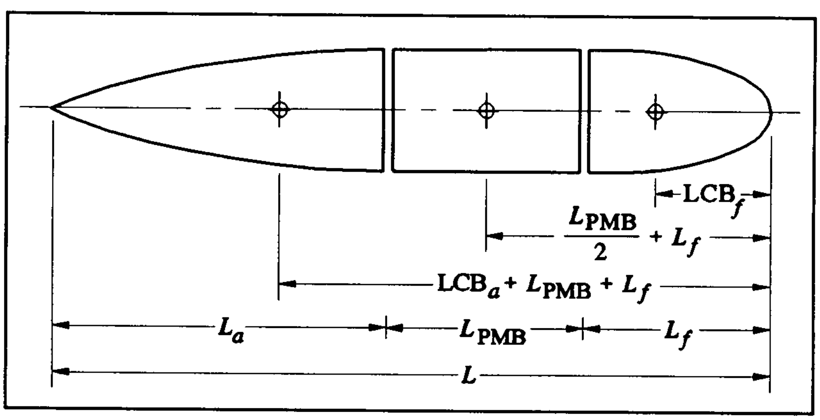


Figure 8. Centers of Various Parts of a Body of Revolution

4. POLYGON

The polygon is a very useful tool for the designer. It determines the size and location of the variable ballast tanks. It is also the ultimate balancing of the weights and buoyancy of the submarine. Referring to Table 2,

$$VL = NSC - Cond\ A \quad (6)$$

$$Mom\ VL = Mom\ NSC - Mom\ Cond\ A \quad (7)$$

The volume of NSC can not change once the submarine is constructed, but the buoyancy can, due to the change in the water density of the ocean. In 1918, The US Navy established the criteria for the variation of the salinity in the ocean. The average in the usual areas of operation was set at 64 pounds per cubic foot or 35 cubic feet per ton. The lightest density was set at 63.6 and the heaviest at 64.3 pounds per cubic feet. With operations in the Arctic a new condition is required, but water density has not yet been established; 63 pounds per cubic foot is suggested for use in the Arctic Ocean. If this criterion is to be met, the size of the variable ballast tanks will have to be substantially increased or other means provided to reduce weight.

The variable load, VL, is made up of three parts. The first is the weight of the variable items that can change from day to day or from hour to hour. The second is the water required to remain in equilibrium or, as referred to, balance the submarine. The third is the residual water that remains in the MBT when they are emptied. There are five extreme conditions of loading. If they all can be accommodated, the submarine is considered to be able to operate in any other condition. The names of these conditions are self explanatory: NORMAL, HEAVY, LIGHT, HEAVY FWD, and HEAVY AFT. Equations 6 and 7 can be expanded to account for the difference in water densities by multiplying NSC by W/64 where W is the weight density of the water under consideration and 64 is the average sea water density in pounds per cubic foot. The water in the MBT can be assumed to have the same density as the water that the submarine is floating in. This is not always the situation, but it can be easily corrected by venting the tanks while submerged. The variable items are usually broken down into the following categories:

- Fixed weights: the weights that can vary but are only slightly changed from patrol to patrol; normal crew is in this category
- Fresh water of various kinds
- Provisions
- Lubricating oil
- Fuel oil and compensating water
- Weapons
- Passengers

Table 2. Weight and Buoyancy Summary

Group #	Weight Long Tons	L Arm Ft	L Mom Ft-Tons	V Arm Ft	V Mom Ft-Tons
1	2888	138.00	398544	15.25	44042
2	907	170.00	154190	14.50	13152
3	209.4	163.00	34132	15.75	3298
4	180.5	70.00	12635	16.25	2933
5	579.8	120.00	69576	15.90	9219
6	256.8	112.00	28762	18.00	4622
7	785.7	60.00	47142	12.00	9428
A-1	5807.2	128.29	744981	14.93	86694
LEAD	1049.322	299.05	313803	15.77	16543
A	6856.522	154.42	1058784	15.06	103237
Var Items	430	132.00	56760	11.40	4902
Var Wat Ball	23.22	130.00	3019	5.00	116
Residual	3.3	125.00	413	2.00	7
NSC	7313.042	153.01	1118975	14.80	108262
MBT	930	51.41	47812	15.80	14694
SUB D	8243.042	141.55	1166787	14.92	122956
Free Flood	389	125.00	48625	14.00	5446
ENV D	8632.042	140.80	1215412	15.83	136645
	BG	1.00	GM	1.03	
	V Arm S	4.00	V Arm M	15.80	
	L Arm S	64954.43	L Arm M	110.00	
	Stab L	3.06	456.52	130.98	59795.6
	Mar L	1046.26			
w = 63.6	410.8135	208.25	Reference for Polygon		130
w = 64.0	456.52	-843.50			
w = 64.3	490.7999	-1632.31			
w = 63.0	342.2537	2837.62			

- Battery electrolyte
- Water in WRT (water-round-torpedo) tanks, torpedo drain tanks, and torpedo tubes

The value of the weight in each category can be estimated from the requirements. The locations can be estimated from the preliminary sketch. The resulting weights and moments can be subtracted using the following equation:

$$\text{Var Load} = \Sigma \text{ Var Items} + \text{Water to Balance} \tag{8a}$$

$$\text{Water to Balance} = \text{NSC x w/64} - \text{Cond A} - \Sigma \text{ Var Items} \tag{8b}$$

(The density of seawater is 64 lb/ft^3.) The difference is the water to balance and its moment about an arbitrary reference. These can be plotted on an abscissa of moment and an ordinate of weight of water to balance. The results will be for five conditions as described above. The plotted locations are referred to as points in the polygon.

The remaining problem is to locate and size the variable tanks that will contain all of the required water and provide the correct moment. The usual tanks are the FWD and AFTER trim tanks near the ends of the submarine and a group of auxiliary tanks near the LCB of the ENVD. If the FWD Trim tank is filled slowly and the weight of water is plotted in tons and its moment in ft-tons from the same reference as the water to balance, it will be nearly a straight line. If the auxiliary tanks are filled in a similar manner, a straight line can be plotted for each auxiliary tank beginning at the end of the FWD trim line. Then as the AFTER trim tank is filled, another line will be developed that will indicate the negative moment. If the tanks are dewatered in the opposite order, a mirror image of the lines will be developed. The result is known as the *polygon*. If all of the points can be located within the polygon, it is assumed that the submarine can be safely submerged in all conditions of loading. Referring to Table 2, it can be seen that there is an interchange between the variable load and the lead. The points can be moved about by changing the weight and location of the lead. Figure 9 is a typical polygon based on a feasibility study of a highly imaginative submarine.

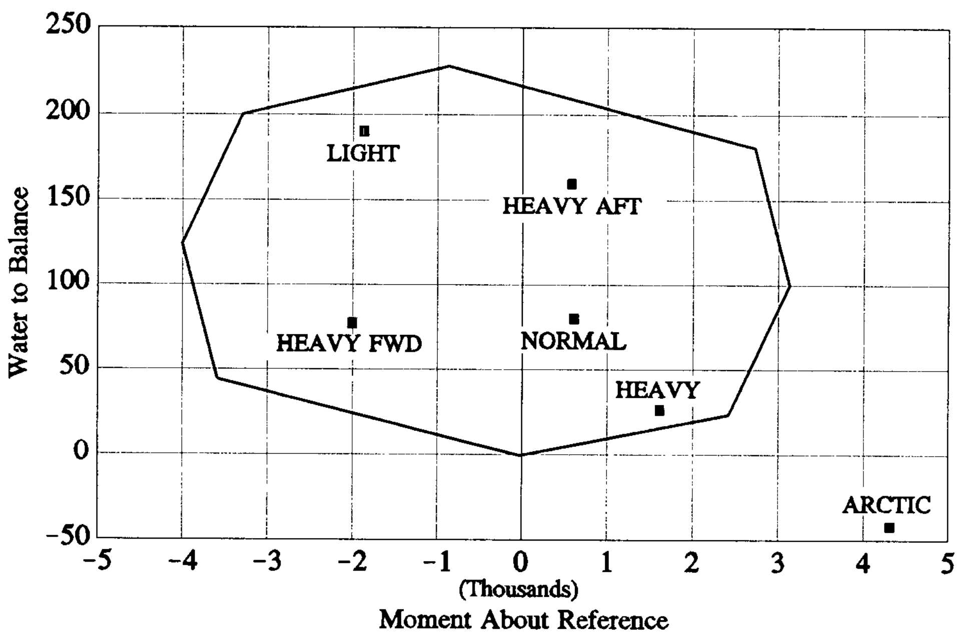


Figure 9. Polygon

5. SPEED AND POWER

At this point, the principal dimensions have been determined and a reasonable balance achieved. These are the foundations needed to estimate the power required to make the specified speed. The methods used are the same as those for surface ships and are contained in many naval architecture textbooks. The following relationship for calculating the effective horsepower was published by Russo *et al* (1960):

$$\text{EHP} = .00872 V_k^3 \left[WS(C_f + \delta C_f + C_r) + A_s C_{Ds} + \Sigma (A_A C_{DA}) \right] \tag{9}$$

WS is the wetted-surface area of the bare hull. V_k is speed in knots. A_s is the total wetted surface of the sail and A_A is the wetted surface of the individual appendages. C_{Ds} is the drag coefficient for the sail and C_{DA} is the drag coefficients of the individual appendages. There are many formulae to calculate C_f. The one most often used was agreed upon at the International Towing Tank Conference,

$$C_f = \frac{0.075}{\log_{10}(RE - 2)^2} \tag{10}$$

where **RE** is the Reynolds number. δC_f is sometimes called the roughness coefficient or the correlation allowance. It is included to cover all of the fabrication uncertainties, fouling of the hull surface, the openings in the hull, and so forth. The value is to be selected from the range of .0003 to .0012. Figure 6 of Arentzen and Mandel (1960) suggests .0004.

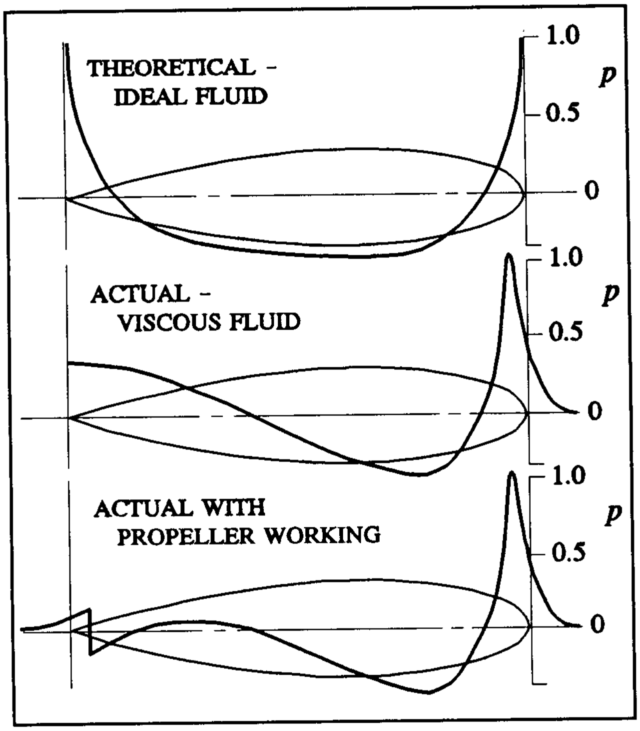


Figure 10. Pressure Distribution Around Hull, Related to Ambient

C_r accounts for the pressure difference along the hull while it is moving. Figure 10 indicates the pressure along the hull as it moves through the water. It is developed from Bernoulli's conservation of energy theorem. The thermal term is ignored as the losses are small, but they are represented by the losses in creating the wake. The upper curve is the classical one that is shown in most text books. The middle curve reflects the loss of energy in the wake due to friction. The lower curve indicates the additional loss of pressure caused by the propeller. It should be noted that there is no negative pressure. Where the curves are below zero, the pressure is less than the ambient. This effect is most notable on the surface at relatively high speeds. The large dip in the water surface just aft of the bow wave is due to the initial low pressure shown in the curves. The additional drag included in C_r may be calculated by integrating the pressure over the surface area of the hull. The result will be a force opposite to the direction of motion. This must be overcome by the thrust of the propeller. Additionally, C_r accounts for the losses due to the turbulence around the appendages and fittings attached to the hull. It is a function of both the maximum cross section and the form of the hull as well as the wetted surface. Considerable data including that listed in Arentzen and Mandel (1960) was plotted and the curve fitted to the following equation, which was partially derived by Jackson (1983):

$$C_r = \frac{.00789}{L/D - K2} \tag{11}$$

For a submarine operating on the surface, an additional wave-making coefficient must be added to Equation 9. Since modern submarines are designed to operate completely submerged, however, the wave making resistance is seldom computed. Another operating condition is snorkeling. This is most important for air dependent submarines. The horsepower required for snorkeling is somewhere between that for the submerged and surfaced conditions.

The fairwater (sometimes called the sail or fin) requires a large fraction of the total power to drive it though the water. Many suggestions have been made to eliminate it, but none of the replacement schemes have proven to be acceptable. The horsepower required is calculated in exactly the same manner as for the main hull. The three drag coefficients are combined into one with the symbol C_{Ds}. Each appendage has similar drag coefficients. They are included in Equation 9 as the summation of A_A times C_{DA}. This too is a large percentage of the hull drag.

The size and shape of the control surfaces is determined by hydrodynamic considerations. For concept design studies of modern submarines, a first approximation is that the sum of $A_A{*}\ C_{dA}$ is equal to $L{*}D/1000$. These terms are all included in Equation 9. With the information above, it is possible to calculate and plot a curve of effective horsepower, EHP, *vs* speed in knots. The same coordinates may also be used to plot SHP vs speed in knots.

6. PROPELLERS

A propeller is a device to convert the torque of the engines into the thrust on the hull needed to overcome the resistance of the submarine. A propeller system can have one or more propellers. Also it can have controllable pitch or counter-rotating propellers. Ducted propellers are now frequently considered. There are advantages and disadvantages of each configuration. The one that has most often been selected for a modern submarine with a body of revolution hull is a single propeller on the axis, aft of the control surfaces. A propeller may have any number of blades. The most efficient will have two blades, but other considerations such as noise signature and strength will favor a higher number of blades. So far the maximum number has been seven. A propeller is designed from data obtained for towing-scale models in the towing tank or in the propeller tunnel. The data from these tests is known as the open water tests as there is no interference from extraneous things such as the hull. There are four major parameters for the recording and use of the data collected. They are:

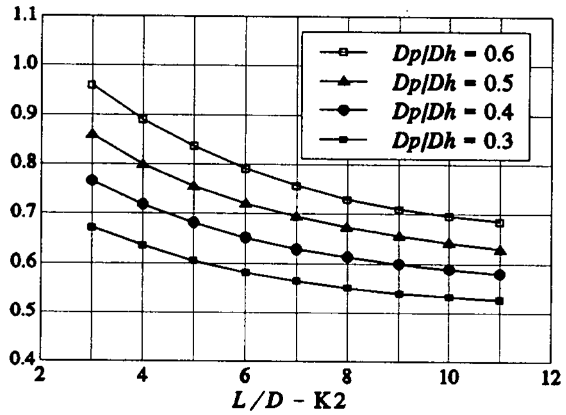


Figure 11. (1 - *w*) *vs* (*L*/*D* - K2)

$$K_t = \frac{T}{\rho d_p^4 n^2} \tag{12}$$

$$K_q = \frac{Q}{\rho d_p^5 n^2} \tag{13}$$

$$J = \frac{V_A}{n\, d_p} \tag{14}$$

$$\eta_0 = \frac{K_t J}{2\pi K_q} \tag{15}$$

where

K_t = thrust coefficient
K_q = torque coefficient
J = advance coefficient
η_0 = open-water efficiency
T = thrust (lbs)
Q = torque (ft-lbs)
d_p = propeller diameter (ft)
n = propeller rate (rev per sec)
ρ = water density (lbs/ft^3)

The data collected from the open water tests can be displayed on propeller charts similar to Figure 14. Propeller characteristics can also be calculated using the Wageningen data given in Lewis (1988), Volume II.

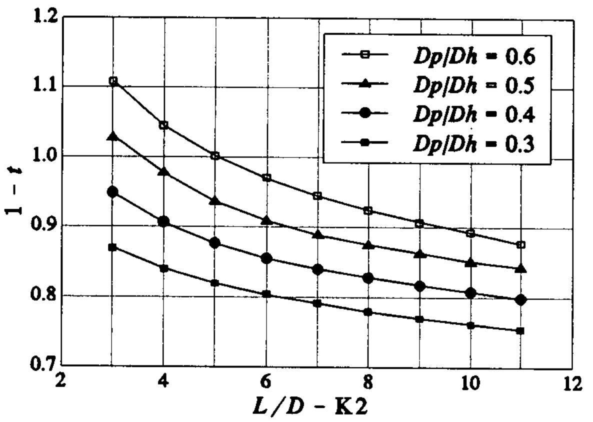


Figure 12. (1 - *t*) *vs* (*L*/*D* - K2)

Wake is defined as a decrease in the velocity of the water along the hull. This decrease becomes larger toward the after end. It is somewhat higher behind the control surfaces, which causes an uneven wake centered around the axis. The propeller on the axis works in this non-uniform wake, which reduces the efficiency and creates noise and vibrations.

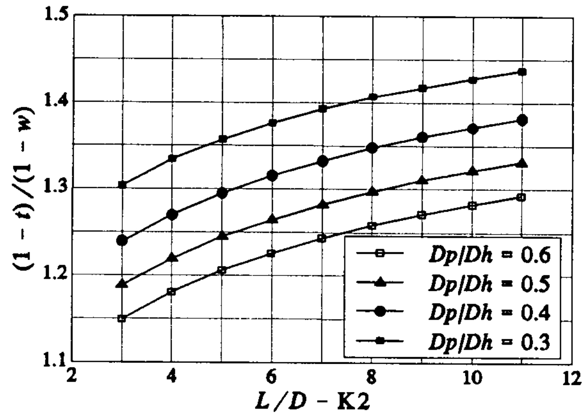


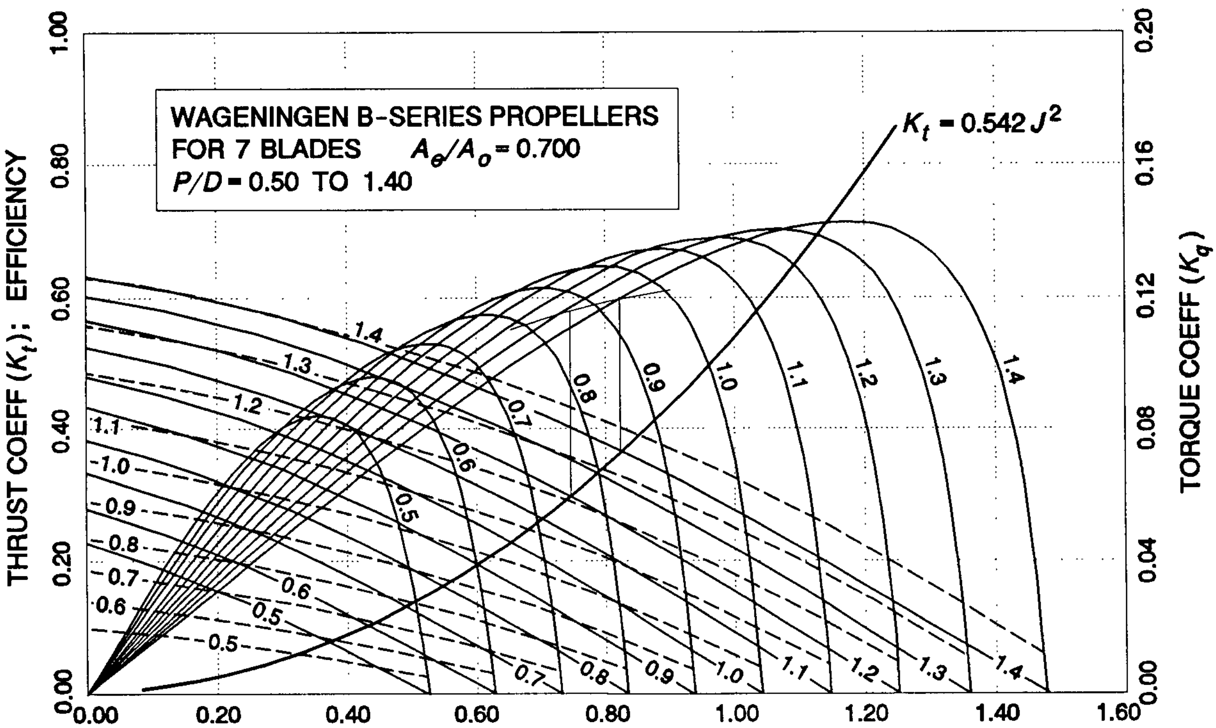
Figure 13. (1 - *t*)/(1 - *w*) *vs* (*L*/*D* - K2)

7. HULL AND PROPELLER INTERACTION

It was stated above that the velocity of the water through the propeller disk was less than the free-stream velocity of the submarine. The fractional velocity change is w, the wake fraction. The velocity at the propeller disk is $V(1 - w)$, where V is the submarine speed.

Another important relationship is that the thrust output of the propeller must be greater than the resistance of the hull. This is due to the reduced pressure ahead of the propeller and in turn on the after end of the hull. The fractional increase in thrust required is t. The thrust required is $R/(1-t)$, where R is the hull resistance. These two factors are difficult to measure either in the model basin or on full size ships. Much of the reliable data is found by full ship-size trials and extracting the data from them. A method for obtaining full size data is outlined by Coxon (1989). Figures 11 and 12 are curves relating $(1-w)$ and $(1-t)$ to (L/D - K2). They have been developed by accumulating data and then curve fitting the scatter. They are sufficiently accurate for use in concept design.

The hull efficiency, which is not a true efficiency as it can be greater than 1, is defined as $(1-t)/(1-w)$. It is related to the amount of energy that can be retrieved from the wake and the increase in the thrust required due to the propeller action on the hull. Figure 13 is a plot of hull efficiency *vs* (L/D - K2).



ADVANCE COEFF (J)

Figure 14. Propeller Thrust and Torque

The hull and the propeller can be combined by the use of the following equation:

$$K_t = \frac{\text{EHP}}{V^3}\left[\frac{550}{\rho D^2 (1-t)(1-w)^2}\right] J^2 \tag{16}$$

V must be in feet per second in order to be dimensionally correct; the factor 550 gives the conversion to units of horsepower. Once EHP *vs* V_k is determined and $(1-t)$ and $(1-w)$ are read from Figures 11 and 12, a curve of K_t vs J^2 can be calculated and superimposed on the open water propeller curve, as shown in Figure 14. This curve will intersect the K_t lines for the various P/D (Pitch/Diameter) ratios. The efficiency of the propeller at the J where the two curves cross can be read off the propeller curve. The P/D ratio and advance coefficient J should be selected where the efficiency is the greatest. Propulsors are now being considered for submarines. Kort nozzles have been installed on tug boats for a good many years as they increase the thrust available from the propeller. They are, however, noisy. Figure 15 indicates the difference between a Kort nozzle and a pumpjet. There is little data available for the design of pump jets, but the procedures would be the same as for an axial flow pump. η_0 can be on the order of .6 to .65.

PC is defined as the ratio of EHP/SHP. It can be calculated using the above information:

$$\text{PC} = \eta_0 \, \eta_h \, \eta_{rr} \tag{17}$$

where η_0 = open water efficiency, $\eta_h = (1-t)/(1-w)$, and η_{rr} = the relative rotative efficiency factor. It is included to account for the fact that the propeller operates in a turbulent wake behind the submarine. When PC is determined, the SHP curve can be added to the EHP curve. Propeller rotational speed can be calculated from Equations (18) and (19). It is of the utmost importance that propeller RPM (revolutions per minute) match the characteristics of the machinery plant.

$$J = \frac{V(1-w)}{nD} \tag{18}$$

$$\text{RPM} = 60\,\frac{V(1-w)}{JD_p} \tag{19}$$

8. STRUCTURES

The structure of a submarine is like any other structure designed to withstand external uniform pressure. The beginnings of technical design were made by the Germans while developing the submarines for WWI. They started by creating a fourth-order differential equation for the deflection curve of the shell and the frames of a ring-stiffened cylinder of infinite length. This eliminated the loading of the ends due to hydrostatic pressure. The purpose of this was to make the basic equation easier to

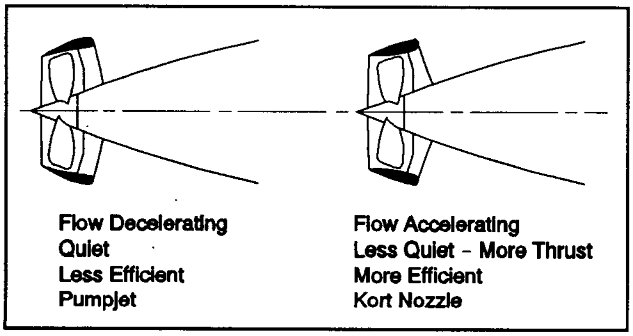


Figure 15. Pumpjet *vs* Kurt Nozzle

solve. The purpose of the reinforcing frames is to keep the shell round. In order to do this, some of the load on the shell must be transferred to the frame by shear at the frame. The amount of the load on the shell that is transferred to the frame has been debated for many years, with no conclusive answer. One can obtain a "feel" for the answer by some simple relationships. The lower part of Figure 16 indicates a typical relationship of the frame and the shell. Let k be the shell load transferred to the frame:

$$\text{frame load} = kPRL_f .$$

Also, let

P = external pressure
R = shell radius
L_f = frame spacing
A_f = area of frame
σ_t = stress in frame
σ_y = yield stress
t = shell thickness

Then $$\sigma_f = \frac{\text{load}}{A_f} = \frac{kPRL_f}{A_f}$$

and $$\frac{kPR}{A_f} = \sigma_y t .$$

Let $$\sigma_y = \sigma_f .$$

Combining, $$\sigma_f = \frac{k\sigma_f t L_f}{A_f} .$$

But $$t L_f = A_s .$$

Therefore $$k = \frac{A_f}{A_s} . \tag{20}$$

k can be plotted for frame shell combinations at various depths. For normal submarine designs, it will be in the range of .375 to .400.

Arentzen and Mandel (1960) and others have shown that submarine hulls can fail in three different modes. They are shell yielding, lobar buckling of the shell between frames, and general instability of the frames and shell between bulkheads or large stiffeners (called king or deep frames). The failure pressure in shell yielding can be determined by the hoop stress formula. When allowable stress can be slightly higher than the yield stress. This can be proven by the Mises-Hinckey relationship. Where the axial stress is just one half the hoop stress, as it is in a submarine, the allowable stress can be 1.16 times the yield stress. This is a theoretical relation that will not be maintained as the structure deforms at high stress levels and the axial stresses increase. A safe figure is about 1.06. The actual failure pressures have to be individually calculated however. The pressure at which the structure will fail by yielding can be found by the formula taught in most high school physics:

$$PDL = 2t\sigma_y L \quad \text{or} \quad P_y = \frac{2t\sigma_y}{D} . \tag{21}$$

Lobar buckling is defined as the buckling of the shell between the frames. There is always an even number of lobes, and the number is related to the frame

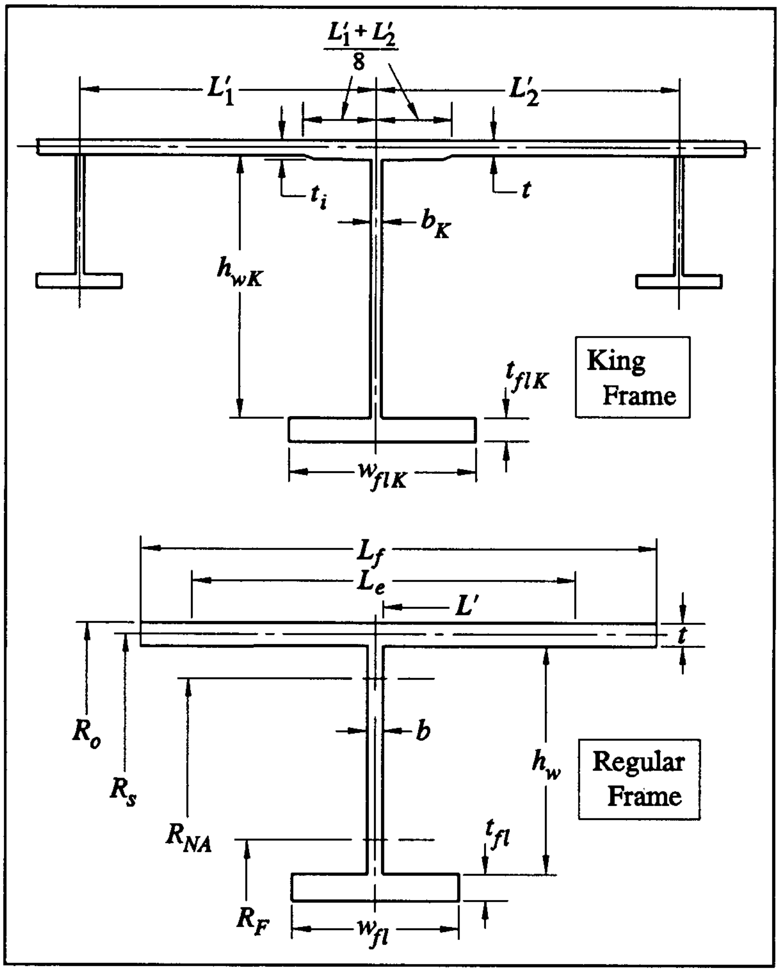


Figure 16. Frame-Shell Relationships

space. The smaller the frame space the larger the number of lobes. von Mises (1933) developed an equation which was the solution of the elastic deflection curve formula. It was quite complicated even though he made some simplification. Windenburg and Trilling (see Trilling (1935)) made some further simplifications, resulting in a formula which is accurate to about $\pm$ 5%. The resulting equation is:

$$P_{cr} = \frac{2.6E(t/D)^{5/2}}{L/D - 0.45(t/D)^{1/2}} \tag{22}$$

where P_{cr} is the collapse pressure (P). One of the assumptions was that the number of circumferential lobes, n, equals $(\pi R)/L_f$. One can see that, as L_f/R becomes smaller, n must become larger. Model tests have proven this to be true. Equation (22) is not acceptable when the denominator approaches zero. One should ensure that it is in an effective range whenever using it. At large L_f/D ratios, Euler's formula can be used:

$$P_{cr} = 2.2E\left[\frac{t}{D}\right]^3 \tag{23}$$

These three equations can be plotted on the same coordinates to create a very informative chart, as shown in Figure 17.

Kendrick (1953) shows that the structure of one frame space can be considered to be made up of two columns, one the shell and the other the frame and the shell of one frame space. The stress in the frame will be the load divided by the area. The other term represents the buckling strength. The resulting equation is:

$$P_{cr} = E\frac{t}{R}\frac{m^4}{(n^2-1+m^2/2)(n^2+m^2)^2} + \frac{(n^2-1)EI_e}{R_f^3 L_f} \tag{24a}$$

where

$$m = \frac{\pi R}{L_B} \tag{24b}$$

(m, the number of longitudinal lobes, should always be near 1) and L_B is the space between bulkheads or king frames. There have been some questions regarding the effective length of the shell when calculating the moment of inertia of the frame shell combination. L_e from Equation (31) should be used when calculating the effective moment of inertia, I_e, for Equation (24a).

Shell Yielding. The axial stress in a cylindrical hull can be shown to be exactly half the hoop stress. van Sanden and Gunther (1952) developed their Equations 92 and 92A from the deflection curve. They reasoned that the maximum stress occurred at the inside surface of the shell, where it bent over the frame, and at the outside surface in midbay. Their Equation 92a gives the tangential stress and Equation 92 the axial stress. The basic parameters are:

R_s = mean radius of the shell

t = shell thickness

σ_x = axial stress

σ_ϕ = tangential stress

$$\theta = \frac{18.2\,L_f/D}{(100\,t/D)^{1/2}}$$

$$\beta = \frac{1.555\,N(R_s t^3)^{1/2}}{A_f + bt}$$

$$B = \frac{bt}{A_f + bt}$$

$$N = \frac{\cosh\theta - \cos\theta}{\sinh\theta + \sin\theta}$$

Equation 92 of von Sanden and Gunther is:

$$P = \frac{2\sigma_x(t/D)}{0.5 + 1.815K[(0.85 - B)/(1+\beta)]} \tag{25}$$

where $K = \dfrac{\sinh\theta - \sin\theta}{\sinh\theta + \sin\theta}$

Their Equation 92a is:

$$P = \frac{2\sigma_\phi(t/D)}{1 + H[(0.85 - B)/(1+\beta)]} \tag{26}$$

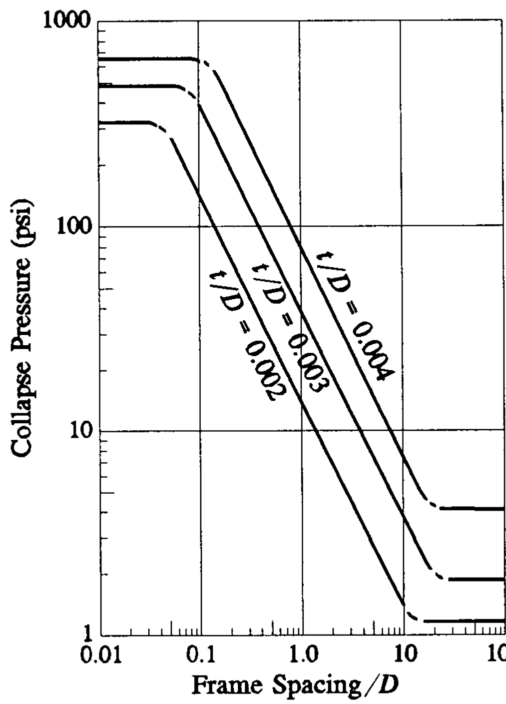


Figure 17. Collapse Pressure

where

$$H = -\frac{3\sinh(\theta/2)\cos(\theta/2) + \cosh(\theta/2)\sin(\theta/2)}{\sinh\theta + \sin\theta}$$

K, *H*, and *N* are transcendental functions that define the bending effect on the shell due to the local framing. *N* is the effect of the deflection of the frame, *H* is the bending effect in the shell at midspan, and *K* is the bending effect of the shell at the frame.

A more exact analysis was made by Salerno and Pulos (1961). Their equation for the deflection curve included an extra term which accounted for the column effect of the end loading. The solution contained a trigonometric function of θ which is the same as for *H*, *K*, and *N*. The solutions included functions F_1 - F_4. As in Equations 92 and 92A of von Sanden and Gunther, the stress at the midpoint between frames and at the frames can be calculated as follows. Let

$\sigma(x_m)$ = axial stress at midbay

$\sigma(x_f)$ = axial stress at frame

$\sigma(\phi_m)$ = tangential stress at midbay

$\sigma(\phi_f)$ = tangential stress at frame

The *o* and *i* symbols denote the outside and the inside of the plates. They are reflected in the following formulae by the + and - signs, the top sign for the outside and the bottom sign for the inside. Then

$$\sigma_i^o(x_m) = \frac{PR}{t}[0.5 \pm aF_4] \tag{27}$$

$$\sigma_i^o(x_f) = \frac{PR}{t}[0.5 \pm aF_3] \tag{28}$$

$$\sigma_i^o(\phi_m) = \frac{PR}{t}[1 - a(F_2 \pm 0.3F_4)] \tag{29}$$

$$\sigma_i^o(\phi_f) = \frac{PR}{t}[1 - a(1 \pm 0.3F_3)] \tag{30}$$

$$L_{eff} = L_f F_1 + b \tag{31}$$

where

$$a = \frac{(1 - \nu/2)}{1 + (bt/A^*) + L_f t F_1/A^*}$$

$$A^*_{\text{external}} = \left|\frac{R_o}{R_{cg}}\right|^2 A_f$$

$$A^*_{\text{internal}} = \left|\frac{R_o}{R_{cg}}\right| A_f$$

$$F_1 = \frac{4}{\theta}\left|\frac{\cosh^2 n_1\theta - \cos^2 n_2\theta}{\dfrac{\cosh n_1\theta \sinh n_1\theta}{n_1} + \dfrac{\cos n_2\theta \sin n_2\theta}{n_2}}\right|$$

$$F_2 = \left|\frac{\dfrac{\cosh n_1\theta \sin n_2\theta}{n_2} + \dfrac{\sinh n_1\theta \cos n_2\theta}{n_1}}{\dfrac{\cosh n_1\theta \sinh n_1\theta}{n_1} + \dfrac{\cos n_2\theta \sin n_2\theta}{n_2}}\right|$$

$$F_3 = \left[\frac{3}{1-\nu^2}\right]^{1/2}\left|\frac{\dfrac{\cos n_2\theta \sin n_2\theta}{n_2} - \dfrac{\cosh n_1\theta \sinh n_1\theta}{n_1}}{\dfrac{\cosh n_1\theta \sinh n_1\theta}{n_1} + \dfrac{\cos n_2\theta \sin n_2\theta}{n_2}}\right|$$

$$F_4 = \left[\frac{3}{1-\nu^2}\right]^{1/2}\left|\frac{\dfrac{\cosh n_1\theta \sin n_2\theta}{n_2} - \dfrac{\sinh n_1\theta \cos n_2\theta}{n_1}}{\dfrac{\cosh n_1\theta \sinh n_1\theta}{n_1} + \dfrac{\cos n_2\theta \sin n_2\theta}{n_2}}\right|$$

$$n_1 = 1/2(1-\gamma)^{1/2}; \qquad n_2 = 1/2(1+\gamma)^{1/2}$$

$$\gamma = \frac{P}{2E}\left|\frac{R}{t}\right|^2\left[\frac{3}{1-\nu^2}\right]^{1/2}$$

ν is Poisson's ratio. σ_h can be substituted for PR/*t* where *P* = pressure at operating depth and $R = R_o$, the outside radius. However this stress must be less than .75 times the yield stress of the metal. This is a different method of providing a factor of safety.

Factors of safety on the pressure at the operating depth are applied by all navies. Some common ones are 1.5 on yield, 2.25 on lobar bucking, and 3.75 on general instability. The higher factors are applied to the buckling modes, which are affected by construction imperfections and residual welding stresses. Fatigue is the primary concern.

Effect of Out of Roundness. The above formulae assume that the shell is perfectly round. Modern fabrication techniques maintain excellent circularities. Practical shipyard considerations, however, make this assumption well nigh impossible to achieve. The following equations take account of the deviation of the shell from the mean circle, which is labeled e_o in this discussion. An inward deflection is positive, and an outward deflection is negative. In the following, it is assumed that ν = .3.

$$\frac{\sigma_i^o(x_m)}{\sigma_u} = \frac{1}{2} \pm aF_4\left[1 + \frac{4Re_o(\alpha+1)}{0.85L^2\alpha}\right] \tag{32}$$

$$\frac{\sigma_i^o(\phi_m)}{\sigma_u} = 1 - aF_2 + \frac{4Re_o}{L^2}\left[1 - \frac{(\alpha+1)F_2}{\beta + (1+\beta)F_1}\right]$$

$$\pm 0.3 a F_4 \left[1 + \frac{4 R e_o (\alpha + 1)}{0.85 L_f^2} \right] \quad (33)$$

$$\frac{\sigma_i^o(x_f)}{\sigma_u} = \pm \frac{1}{2} a F_4 \left[1 + \frac{4 R e_o (\alpha + 1)}{0.85 L_f^2} \right] \quad (34)$$

$$\frac{\sigma_i^o(\phi_f)}{\sigma_u} = 1 - a \left[1 + \frac{4 R e_o (1 - \beta)(1 - F_1)}{0.85 L_f^2 \alpha} \right]$$

$$\pm 0.3 a F_3 \left[1 + \frac{4 R e_o (\alpha + 1)}{0.85 L_f^2 \alpha} \right] \quad (35)$$

where

$$\alpha = \frac{A^*}{L_f t} \qquad \beta = \frac{b}{L_f} \qquad \sigma_u = \frac{P R_s}{t}$$

$$w_{max} = \frac{\sigma_u R_s}{E} \left[1 - \frac{\nu}{2} - a F_2 \right]$$

w = deflection along the curve of the shell

Frame Strength. The strength of the frames is very important. If they are too weak, the hull will fail prematurely. If they are too strong, the hull will be too heavy and they will overplay their part: they will not deflect enough and will thereby cause the shell to fail at some lower pressure. There are both compressive and bending stresses in the frames. The compressive stresses can be calculated using the works of von Sanden and Guenther: $q = FP_c$, where q is the load per inch of radius of the frame. F (see below) is related to the length of the shell included in the calculation, and P_c is the design pressure. If $\nu = 0.3$, and the other terms are as previously defined,

$$F = b \frac{1 + 0.85 (\beta / B)}{1 + \beta}$$

$$\sigma_c = \frac{F P R_{cg}}{A_f + bt} \quad (36)$$

Kendrick (1953) developed the following equation for determining the bending stress in the frame:

$$\sigma_b = \frac{E c e_o (n^2 - 1)}{R_{NA}^2} \frac{P_c}{P_{cr} - P_c}$$

where c is the distance from the neutral axis (NA) to the outermost surface of the frame, P_c is the design collapse pressure, P_{cr} is the collapse pressure due to general instability, and E is Young's modulus. The total stress in the frame is the sum of the compressive and the bending stresses:

$$\sigma_T = \sigma_c \pm \sigma_b \leq \sigma_Y \quad (37)$$

King-Frame Strength. The strength of the king or deep frames can be found in the same manner as for the normal frames, the only difference being the length of the shell included in the formula. The king frame will start to fail in the same mode as the general instability. Since the king frame is very rigid, an insert must be placed in the shell at the frame. The length should be about half the normal frame space. The thickness t_i is on the order of 1.3 times the thickness of the shell. D_K = the diameter of the CG of the king frame. L_B is the length of the shell between king frames and has a value of about πR_s (or $\pi D_s / 2$).

P_{cr} = lobar buckling failure

$$\sigma_c = \frac{F_{KF} P R_{KFcg}}{A_{KF} + b_{KF} t_i}$$

$$F_{KF} = b_{KF} \frac{1 + 0.85 \beta_{KF} / B_{KF}}{1 + \beta_{KF}}$$

$$B_{KF} = \frac{b_{KF} t_i}{A_{KF} + b_{KF} t_i}$$

$$\beta_{KF} = \frac{1.555 (R_o t_i^3)^{1/2}}{A_{KF} + b_{KF} t_i}$$

$$\sigma_b = \frac{E c e_o (n^2 - 1)}{R_{NA}^2} \frac{P_c}{P_{cr} - P_c}$$

The total stress in the king frame is the sum of the compressive and bending stresses:

$$\sigma_T = \sigma_c + \sigma_b \quad (38)$$

End Closures. There are many types of end closures. Some are stiffened flat plates and others are complete or partial hemispheres. The hemi-ellipse is often used. Additionally, combinations of the above with cones are used where the hull diameters are large. Since most end closures have many penetrations, which make the computations difficult, the thickness may be determined by empirical formulas. There is one for the compressive and one for the bending or buckling modes of failure. The formulas for hemispheres are as follows:

$$\sigma_c = \frac{P R_o^2}{2 t R_1} \quad (39)$$

where R_1 = radius to midplane over a critical length, which, for steel, is $L = (2.42 R_1 t^{1/2})$

$$\sigma_{avg} = 0.84 E \left[\frac{t}{R_o} \right]^2 \quad (40)$$

The stresses calculated should both be equal to or less than the yield stress of the metal. The greater thickness calculated by the two formulae should be the one selected.

Bulkheads. Bulkheads are very heavy. They too can be made of stiffened flat plates or portions of spheres. It is desirable to design them to withstand flooding at the collapse pressure of the hull. For most deep diving submarines, this results in an almost prohibitive weight penalty. Some compromise is usually made after evaluating all of the considerations. There should be at least two escape compartments, which then requires at least one bulkhead that will withstand collapse depth. Usually, there are more that will withstand a pressure of lesser depth. The procedure for design is the same as for any structure that has to withstand pressure on one side. The most important factor is to provide sufficient strength to transfer the load on the bulkhead into the shell. The weak point is the shear stress at the shell. Finite element analysis is the best approach. The number of bulkheads and their strength is a matter for much soul searching in the development of the concept design.

Deflection. The deflection due to an increase in stress is very important in all phases of the design process. It causes movement and stress in piping systems, components, foundations, and many other places. Means of accommodating these movements and stresses must be provided. If not, very serious casualties could occur that could hazard the submarine. A simple relationship of stress and deflection is:

$$\frac{\delta R}{R} = \frac{\sigma}{E} \tag{41}$$

It is important to note that the deflection is due to the stress level in the metal. The stress level is in turn due to the loading on and the scantlings of the structure.

9. HYDRODYNAMICS

The hydrodynamics of submarines is divided into several major parts. One is dynamic stability and another is controllability. A third is the resistance when being propelled through the water. They are all related but may be considered separately.

When submerged, a submarine must operate in a set of coordinates established by the earth's geometry. Another set of coordinates it that fixed to the submarine itself. The two can be combined by the use of Euler angles, as described in Clayton and Bishop (1982). These relationships are rather complex, but necessary when predicting trajectories of the submarine in space. For the purpose of concept design, it is sufficient to consider only the vertical and horizontal planes. In most operations, the submarine follows a path consisting of straight lines connected by horizontal turns and vertical movements. While discussing hydrodynamics, the special nomenclature outlined by Comstock (1967), pp. 461, 604-606, and 717, will be used. The basic formulae for hydrodynamics are developed from Bernoulli's conservation of energy equation. Force and moment components are given by:

$$F = C_{(\,)}\frac{1}{2}\rho A V^2 \tag{42}$$

$$M = C_{(\,)}\frac{1}{2}\rho A V^2 x \tag{43}$$

The subscript parentheses on C indicate that there are different coefficients for the various force and moment components, such as lift and drag. In some cases, the area, A, is the wetted surface and in others it may be the maximum cross-sectional area. In order to avoid ambiguity, $A = L^2$ for force components and $Ax = L^3$ for moment components . Six equations can be developed from Newton's laws of motion, where force = ma = mass × acceleration and moment = $I\alpha$ = moment of inertia × angular acceleration. The six equations result from two equations in each of the three planes. The forces and moments are all hydrodynamic in nature. The other side of the equations are related to mass and geometry. The linear and rotational accelerations are the result of the hydrodynamic forces.

There are two conditions that must be considered, and they are interrelated. The first is the controls-fixed condition, in which the control surfaces are assumed to be fixed at a zero angle of rotation. This is the primary condition while steaming on a straight course. When the control surfaces are operating, they can override or reverse the effects of the surfaces in the fixed condition.

Land (1916) presented a very simple and unsophisticated explanation of the stability of submarines. He pointed out that, while proceeding on a straight path, there are no transverse forces. However, if the submarine is disturbed linearly, rotationally, or both, in a manner to create an angle of attack between the flow of the water and the submarine, a force and a moment will be generated. A first consideration will be in the horizontal plane. The force will act somewhere near the quarter length of the outer hull. This will cause a moment equal to the force times the distance to the center of gravity. The force and moment in turn will cause linear and rotational accelerations not only of the submarine but also of the surrounding water. The inertia effects of the submarine and the surrounding water create an additional force and moment on the submarine. If these are opposite and greater than the force and moment created by the disturbance, the submarine will return to a straight line course, which will be slightly different from the original. If this happens, the submarine will be hydrodynamically stable. If not, the force and moment will be disruptive and the submarine will be unstable. It will then follow a circular path. In the vertical plane, the submarine will follow a sinusoidal path which may decrease or increase in magnitude, depending on the static stability.

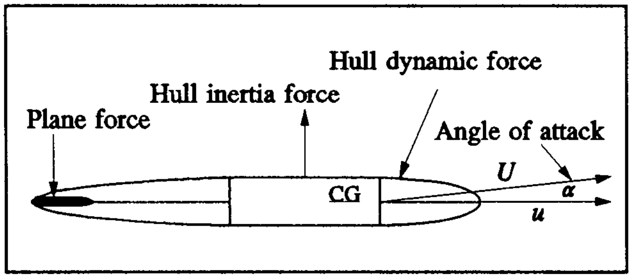


Figure 18. Fixed-Planes Forces

A big factor in ensuring that the submarine will be stable is the size and location of the after control surfaces. In order to be most effective, the rudders and stern planes should have a high aspect ratio. This is because the slope of the curve of lift coefficient *vs* angle of attack is steeper with a high aspect ratio. Figure 18 indicates the primary forces with controls fixed.

When the control surfaces are moved, they change their effect on the submarine and cause it to turn in a controlled manner. The size, shape, and location of the after control surfaces must be such that both conditions can be satisfactorily met. Figure 19 will help to make the movable-plane condition more clear.

The vertical plane condition is quite similar, but has an extra term due to the static stability, described in terms of BG. This is always a stabilizing moment, which is proportional to the sine of the trim angle. It might be called the ultimate safety factor, as it is most effective at low speeds and can be brought into play by slowing down.

All of the other terms in the equations are proportional to V^2. At higher speeds, they are much stronger than the static moment and so override its effects. The hull of a submarine is a lifting body, although a very poor one. It may have a lift force and a moment tending to raise the bow. This may be overcome by putting a dive angle on the stern planes or negative angle of attack on the hull. Sometimes it is overcome by creating an opposite force and moment by shifting variable ballast. This is considered to be poor practice as the submarine will not be in static trim. If power is lost, excessive trim angles may result, with the weight being greater than the buoyancy, which could be a hazard to the submarine.

Snap roll is a phenomenon in the transverse plane. Snap roll has been known since the big airships. It was confirmed in submarines when the ALBACORE first went on sea trials. All submarines heel inboard in the initial phases of high speed turns. When starting into a turn, the hydrodynamic forces are large and act above the center of gravity. This causes a moment that makes the submarine roll towards the center of the turn. As the submarine continues its turn, the angle of attack on the forward end is reduced by the combination of the transverse and forward velocity vectors. In turn, the roll moment is reduced, and the submarine soon reaches a steady state in which the turn rate and the roll angle are constant. The snap roll can be quite large, sometimes fifty percent greater than the steady roll. As the submarine rolls in the turn, the rolling forces are diminished by the $\cos^2$ of the roll angle. Figure 20 demonstrates the principles involved. The longitudinal position of the fairwater has a big impact on the roll. Most sails are quite well forward, for purposes relating to internal arrangements. At the initial stages of the turn, a rather high angle of attack is generated due to the cross flow over the fore end of the submarine. This in turn can produce a bow-down force and, when combined with other hydrodynamic forces, cause the submarine to acquire a large down angle. If prompt action is not taken to correct the situation, large depth excursions can result.

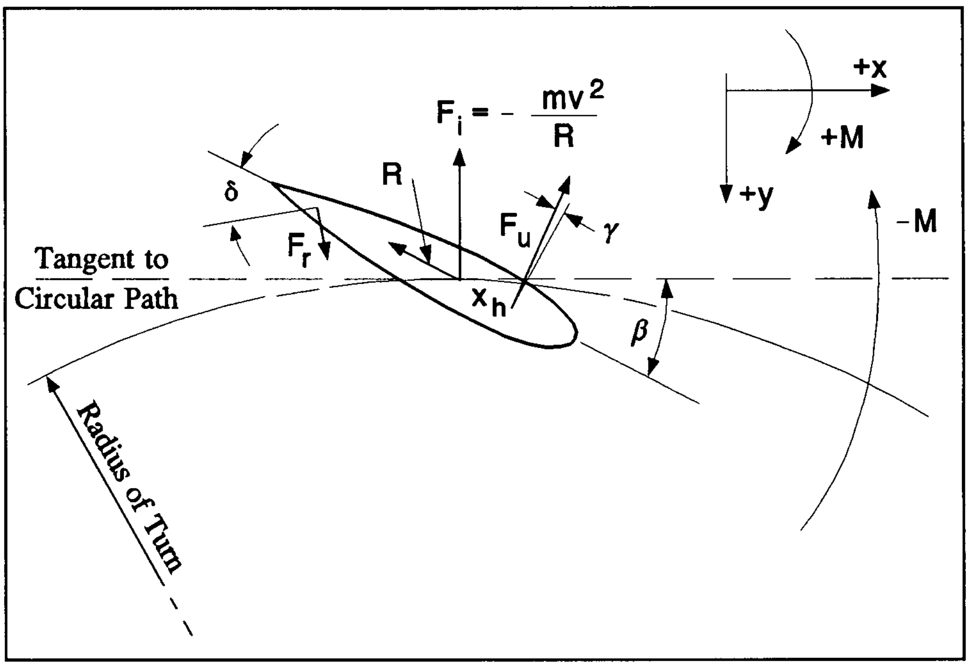


Figure 19. Hydrodynamics Forces on a Turning Submarine

There are two locations of importance in the understanding of hydrodynamics of submarines, the neutral point and the critical point. The neutral point is the location where an applied force will change the depth but not the trim angle. The critical point is the location where an applied force will change the trim angle but not the depth. The location of the neutral point is always forward of the center of gravity and does not change much with speed. On the other hand, the distance of the critical point from the neutral point is inversely proportional to the square of the velocity. At low speed, it is near the position of the control surfaces. At extremely high speeds, it is near the center of gravity. On most modern submarines, it is about half the distance between the center of gravity and the after control surfaces at full speed. Figure 21 will help visualize the location of these forces.

Since the effect of the stern plane is proportional to V^2, at low speeds it is insufficient to create a moment that is greater than the static stability. Therefore a rise angle on the plane will cause a net downward force, and the submarine will go deeper. The converse is true when a dive-plane angle is applied. This is of little concern in large submarines, as they seldom operate at such low speeds. It is very important for small submersibles, as they frequently operate at speeds in the range of concern. This is sometimes referred to as the plane-reversal syndrome.

Arentzen and Mandell (1960) included the solution to the differential equation that is the end point of the equations of motion. If the various coefficients are known, the term and exponent coefficients can be determined for the equation. They show that if these have the proper sign the submarine will be either stable or unstable to various degrees.

The size and location of the control surfaces are most important in the stability and control of any ship. They are more important for submarines, which have to operate submerged and on the surface. This creates many

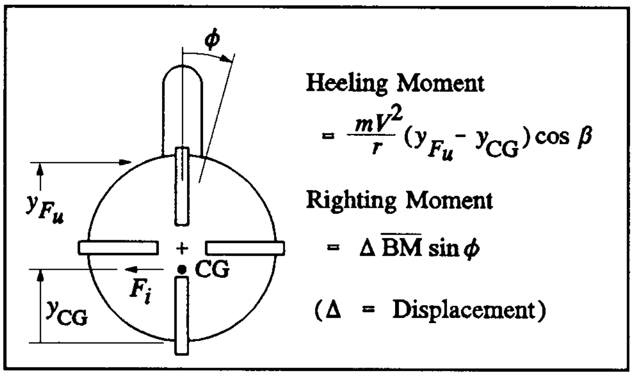


Figure 20. Heel Due to a Turn

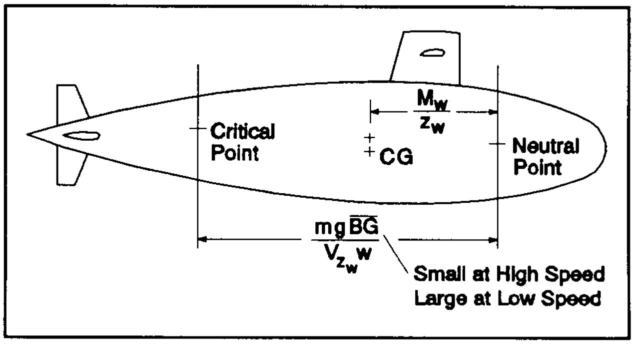


Figure 21. Critical and Neutral Points

problems for the designer. An example is that, if the rudders are properly sized for submerged operations where both a bottom side and top side rudder can be installed, the topside rudder may come out of the water while on the surface. Such a situation may cause undesirable characteristics while on the surface. Diving planes are not required on the surface so they can be optimized for submerged operation.

Rudder sizes and location can be selected such that the inertial forces are opposite and greater than the hydrodynamic forces in the fixed-plane condition. If they meet that criterion, they will probably be adequate for the control of the submarine. If one has the proper hydrodynamic coefficients, it is possible to choose the sizing and location of the control surfaces. Sometime during the later stages of design, it will be necessary to determine the coefficients by calculation or by tests in the model basin. These should be deferred until after the concept design is quite well established. Table 3 provides a means of obtaining a first approximation of the size of the control surfaces based on previous successful designs. The projected area of the control surfaces is equal to the number of surfaces times the volume of the envelope displacement raised to the two-thirds power times the coefficient in the table.

TABLE 3

SUB TYPE	R	B	S	F
SLOW ATTACK	.07	.03	.16	.04
FAST ATTACK	.07	.03	.10	.04
MISSILE	.09	.05	.10	.06

10. ARRANGEMENTS

The arrangement of systems, components, stores, and people is much more difficult in a submarine than in any other vehicle. The volume of the hull of the submarine is fixed by the weight of the submarine. If more volume is mandatory, it can only be provided by making the submarine larger, but this will increase the amount of lead to be carried and reduce the speed if the same power is provided. If the power is increased in order to meet the speed requirements, the submarine will grow even larger. The skill and experience of the designer is put to a crucial test in making a satisfactory design. It is of the utmost importance that the weight and buoyancy match, as well as the LCG and LCB. This is demonstrated in Table 2. Next, VCG must be a specified distance below the VCB, and the LCG and the LCB must be equal both on the surface and submerged.

Systems in submarines are made up of a great many pipes, components, wires, electrical and electronic equipment, crew spaces, stores, and so forth. The heavy weights are driven towards the LCG. It is desirable to put the sonar sensors in the forward end, which is also a favorite location for the weapon discharge system. Main propulsion and other rotating machinery are placed as far as possible from the sonar sensors and so tend to be in the after end. There has to be some compromise. Equipment has to be located so that the items buried outboard of them can be maintained and repaired. Air conditioning equipment must be isolated and silenced. The ducts to and from remote compartments are large and awkward to install. Privacy for the crew is difficult at best. It is most important that attention be paid to providing the best compromises. A long patrol inside an iron cylinder becomes boring after the first couple of months. There is much tradition and past experience involved in making the arrangements acceptable, and they should not be taken lightly.

Periscopes have long been a key consideration in the arrangements as they penetrate all deck levels. The control room of the submarine is located around the periscope stand. It is quite possible that electronic periscopes will be accepted in the newer designs, which will make improvements in arrangements and interior communications.

Many of the components are long and so have to be located parallel to the axis of the pressure hull. This results in a minimum length of the hull to accommodate the items when placed end-to-end. This minimum length is known as the "stack length."

Access for the loading of stores, weapons, spare parts, *etc.*, demand openings in the pressure hull. They are also required for the access and escape of the personnel. There must be a compromise between the desire to have many openings and the absolute minimum required.

11. SUMMARY

The concept design of a submarine is a very complex undertaking, and it is mandatory that the designer have a very inclusive understanding of the interrelations of all the various features and the proposed operations of the submarine. The first requirement is that the submarine be functional and reliable. By its very nature, it spends much of its life operating in a very hostile environment. Included in this paper are some basic considerations and helpful concepts for a first approach towards a concept feasibility study. Caution should be used when trying to apply the data included in the text. It is intended that the paper give an insight into the complexity of the problem. There is much more to the design of a submarine, but the above outlines a beginning concept. The entire design process is not something that can be learned in a weekend.

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APPENDIX: ENTRANCE AND RUN OFFSETS

Table 4. Offsets for Bodies of Revolution

ENTRANCE: $y_f/(D/2)$ [Equation (2)]

x_f/L_f	n_f = 2.0	2.5	3.0	3.5	4.0	4.5
0.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.100	0.9950	0.9987	0.9997	0.9999	1.0000	1.0000
0.200	0.9798	0.9928	0.9973	0.9990	0.9996	0.9998
0.300	0.9539	0.9800	0.9909	0.9958	0.9980	0.9990
0.400	0.9165	0.9582	0.9782	0.9883	0.9935	0.9964
0.500	0.8660	0.9251	0.9565	0.9739	0.9840	0.9900
0.600	0.8000	0.8774	0.9221	0.9490	0.9659	0.9768
0.700	0.7141	0.8098	0.8693	0.9079	0.9337	0.9514
0.800	0.6000	0.7119	0.7873	0.8395	0.8766	0.9036
0.850	0.5268	0.6448	0.7280	0.7877	0.8315	0.8643
0.900	0.4359	0.5570	0.6471	0.7146	0.7658	0.8054
0.950	0.3122	0.4287	0.5225	0.5969	0.6563	0.7040
0.960	0.2800	0.3933	0.4867	0.5621	0.6230	0.6726
0.970	0.2431	0.3516	0.4437	0.5196	0.5820	0.6334
0.980	0.1990	0.2999	0.3889	0.4644	0.5278	0.5811
0.990	0.1411	0.2280	0.3097	0.3824	0.4455	0.5001
0.992	0.1262	0.2086	0.2877	0.3590	0.4217	0.4762
0.994	0.1094	0.1861	0.2615	0.3309	0.3927	0.4471
0.996	0.0894	0.1583	0.2286	0.2949	0.3551	0.4089
0.998	0.0632	0.1200	0.1816	0.2421	0.2988	0.3508
1.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
C_{pf}	0.6667	0.7500	0.8061	0.8455	0.8740	0.8944
C_{wsf}	0.7854	0.8452	0.8833	0.9089	0.9270	0.9476
LCB_f/L_f	0.6250	0.5955	0.5755	0.5612	0.5507	0.5437

AFTER RUN: $y_a/(D/2)$ [Equation (3)]

x_a/L_a	n_a = 2.0	2.5	3.0	3.5	4.0	4.5
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	0.9900	0.9968	0.9990	0.9997	0.9999	1.0000
0.20	0.9600	0.9821	0.9920	0.9964	0.9984	0.9993
0.30	0.9100	0.9507	0.9730	0.9852	0.9919	0.9956
0.40	0.8400	0.8988	0.9360	0.9595	0.9744	0.9838
0.50	0.7500	0.8232	0.8750	0.9116	0.9375	0.9558
0.60	0.6400	0.7211	0.7840	0.8327	0.8704	0.8996
0.70	0.5100	0.5900	0.6570	0.7130	0.7599	0.7991
0.80	0.3600	0.4276	0.4880	0.5421	0.5904	0.6336
0.90	0.1900	0.2316	0.2710	0.3084	0.3439	0.3776
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
C_{pa}	0.5333	0.5952	0.6429	0.6806	0.7111	0.7366
C_{wsa}	0.6667	0.7143	0.7500	0.7778	0.8000	0.8359
LCB_a/L_a	0.3125	0.3333	0.3500	0.3636	0.3750	0.3949

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APPENDIX: ENTRANCE AND RUN OFFSETS

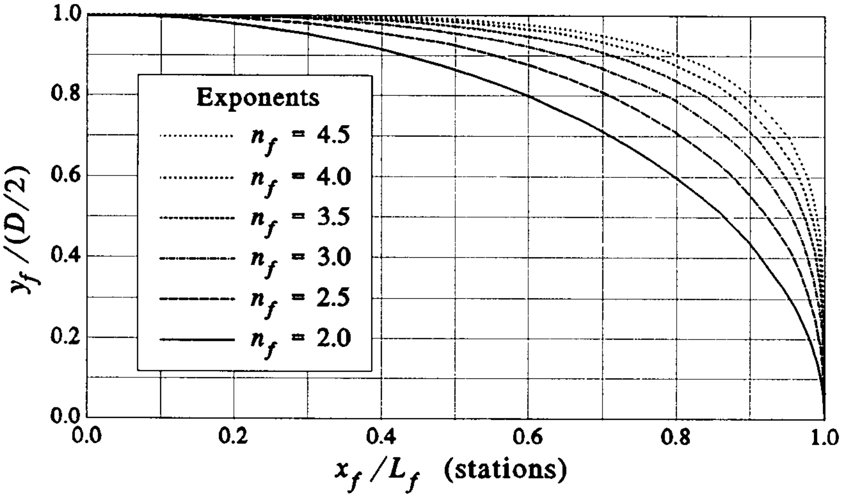

Figure 22. Entrance Offsets (Equation (2))

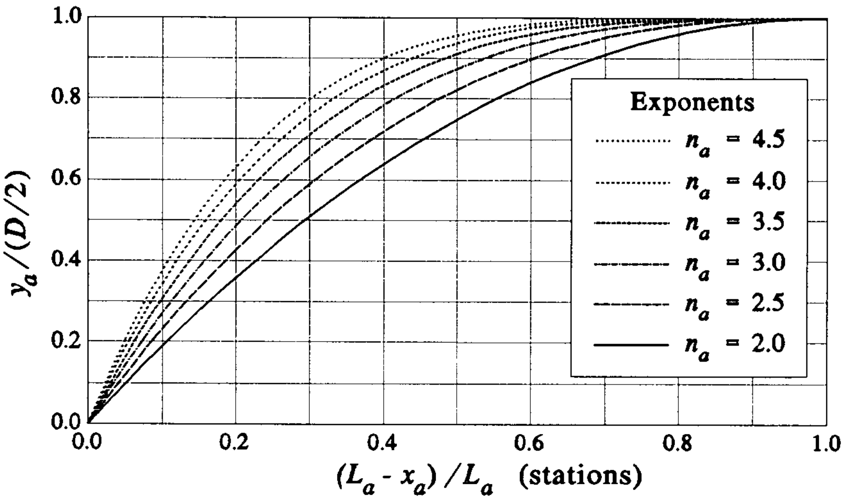

Figure 23. Run Offsets (Equation (3))

[*Discussion and Closure follow*]

Discussion

Carl D. Fast, Member

I would like to thank the author for presenting a wealth of current and useful information in one location. It seems as if papers which discuss submarine design, such as this one, are presented only once in a generation. I am sure it will benefit the naval architecture community and hopefully dispel some of the mystique surrounding submarines. After all, they are just crowded, round boats that sink on purpose.

In discussing so many aspects of the submarine concept design process it is impossible to cover all areas in detail. There is one step, however, worth expounding which is of great importance in establishing a submarine concept; i.e., choosing a hull diameter. This single step affects the entire design and is necessary before any calculations can proceed.

A quick survey of existing submarines (see Fig. 24 herewith) shows some definite trends and provides some insight into the diameter decision. First, the range of diameters is primarily dependent on the number of decks and displacement. Also, the increment of diameter with number of decks is roughly seven to eight feet with a wider variation in smaller boats. In high level concept studies this may provide a sufficiently detailed approach to diameter selection. However, selecting the optimum diameter for a specific concept can result in more arrangeable space for a given displacement or a reduced displacement.

As the concept matures additional factors should be considered in this decision. Diameter drivers include the propulsion plant, weapons stowage, deck area and internal volume requirements for variable ballast, batteries, fuel or air-independent system elements. Limiting factors include draft restrictions and the need to minimize displacement, especially in stackup length governed designs. Optimizing the diameter will lead to a minimum displacement while maintaining adequate volume and deck area.

Some arrangement parameters involving diameter are shown in Fig. 25 for typical two- and three-deck submarines. A minimum clear headroom requirement sets the limits of usable manned areas on upper decks while the framing intersection with the tank top sets lower deck width. A reasonable value for clear headroom is 6 ft-3 in. Areas outboard of the clear headroom limits are assumed to be filled with electronics, stowages, and the many required through systems and services.

Deck spacing can range from seven to eight feet depending on headroom, depth of deck structure and the arrangement of services attached under the deck. Recent trends in modular, end-loaded construction techniques favor more deck-mounted services resulting in larger spacing.

The height of the tank top above the baseline is the remaining parameter needed to locate the platform deck heights within the ship. This parameter will also govern available internal tank and battery volumes. Deck heights can also be established using the distance from the first platform to the hull at top centerline, with 9 ft-6 in. to 10 ft-0 in. being a reasonable first estimate. If this method is used care must be taken to account for adequate tank volumes. Disregarding tank volume at this stage of design will result in an artificially low diameter.

The optimum diameter can be determined by maximizing usable deck area for a given displacement at various deck spacings and deck heights. This is easily accomplished using a PC-based spreadsheet which geometrically relates the diameter, tank top or first deck height, deck spacing, and resulting usable deck areas with displacement. Figure 26 shows a typical curve for a fixed-deck spacing and height of first platform. Note that the curve is quite shallow at the minimum indicating some flexibility in practical choice of diameter. Also, the smaller diameters which appear more efficient should not be considered due to lack of tank volume. If total deck area and tankage requirements are known based on arrangements or historical data, this spreadsheet can also calculate projected compartment lengths.

Optimum diameters from iterations over a range of deck spacings and tank top heights can then be plotted as shown in Figs. 27 and 28. These should cover the range of variations expected throughout concept development as deck area and tankage requirements are established. These figures represent only one set of assumptions for the various factors which must be included in the calculations. Specific diameter optimization plots should be created for each concept as details are developed.

This approach provides a quick and flexible method to base diameter decisions and to meet the rapidly evolving requirements of a concept design.

Fritz Abels, Visitor, Ingenieurkontor Lübeck, Germany

May I first of all congratulate the author on his excellent paper. The presentation is an exceptional event and will find national and international attention and appreciation.

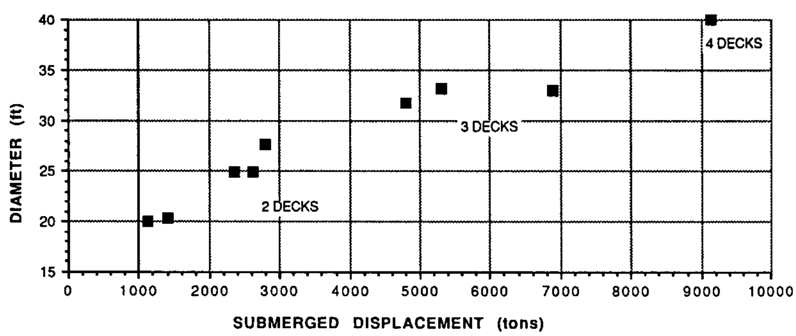

Fig. 24 Typical submarine diameter trends

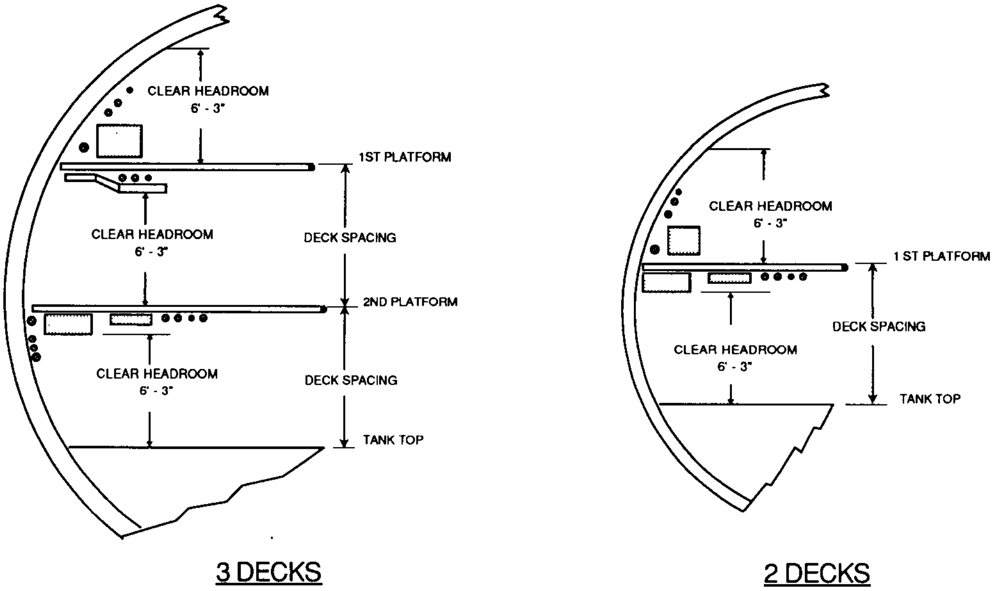

Fig. 25 Typical submarine arrangement sections

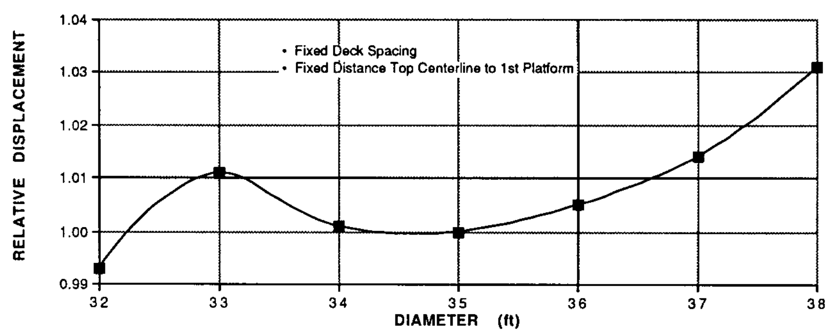

Fig. 26 Typical optimization curve

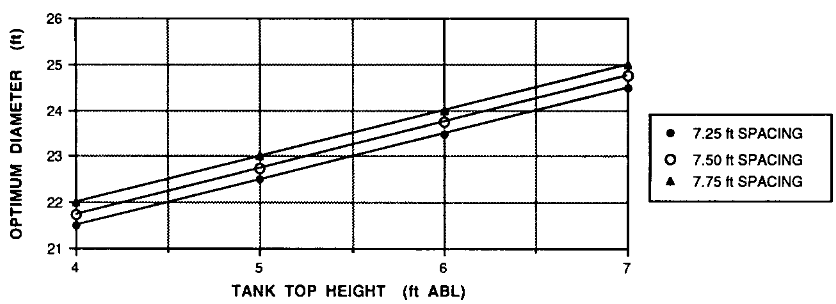

Fig. 27 Optimum diameter curves for two-deck submarines

As vice president of our German Society of Naval Architects and Marine Engineers (STG), as chairman of the German Submarine Design Organization IKL with more than 30 years experience in submarine development and design, as a teacher on this subject at the University of Hamburg, and as a naval architect, it is a great honor for me to discuss this paper.

Within the overall design of submarines, the concept design is an essential step where the boat is developed and the design frozen. The fundamentals of the design are similar worldwide; the procedures, however, sometimes are different. This depends on where the design is conducted.

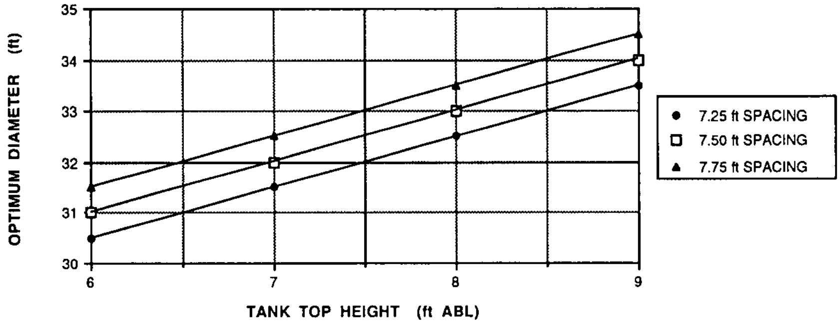

Fig. 28 Optimum diameter curves for three-deck submarines

In studying the paper, I find two essential differences to the procedures in Germany, which are described in a paper by Abels (1992). They are:

1. U.S. Navy submarines are large and nuclear propelled. German boats are small and conventionally driven, though partially air independently propelled in the future.
2. The fundamentals presented here are based on theory, experiment, experience, and on teaching at M.I.T. In our company in Germany, these naval architectural fundamentals are based mainly on computer programs with experience and a database of some 110 submarines built for 16 different navies.

To summarize, it would be a good task for us to extend your valuable diagrams and design procedures to lower numbers for smaller submarines and to compare both of our results of the fundamentals.

Reading the paper was an exciting experience for me. Many interesting subjects are dealt with, which ask for comments and questions. For simplification, only the headings of the different subjects I would like discussed and clarified are listed here:

—Influence of tonnage limitations on the design.
—Demand for low signatures for good stealth capabilities.
—Margin growth in weight and volume, stability and trim.
—Single-hull versus double-hull configuration.
—Trim polygon for small submarines: how many loading conditions are required in practice?
—Influence of HY steel quality on pressure hull weight for lower depths.
—Safety factors for different modes of failure; why not one safety factor against collapse depth?
—Out-of-roundness and tilting of frames; do you consider the tilting?
—Details of hydrodynamics in snorkel condition.
—Dependence of wave resistance on depth.
—Hydrodynamic stability in the horizontal and vertical plane; in the horizontal plane a sinusoidal path is not possible.
—Cross-rudder versus X-rudder.
—Influence of AIP systems.

As you can see, there are many interesting points to discuss. My thanks to the author.

Additional reference

Abels, F., "German Submarine Development and Design," paper presented at the SNAME/ASE Naval Ship Design Symposium, Arlington, Va., Feb. 25, 1992.

R. K. Burcher, Visitor, University College London

Captain Jackson's paper is a welcome contribution to providing an understanding of the involved subject of submarine design. In the limited space available he has given the basic reasons together with many useful formulas and graphical data. I am sure that many students and practitioners of submarine design will be glad to refer to this paper as they have with the much earlier paper by Aretzen and Mandel (Ref 1 of the paper).

The first point of my discussion is how to initiate the concept sizing of the boat. The author has approached this by estimating the weights and thence to volume balance. I would argue that it is preferable to initiate the sizing by a volume estimate and subsequently seek a weight balance. Many of the subsystems and certainly tankage are dependent on the volume of the hull and therefore it is difficult to estimate the weight until the size is known. Whereas a reasonable initial estimate of pressure hull volume can be made from the space demands of the payload (weapons and sensor equipment), accommodation and propulsion plant. Admittedly, the propulsion power is also form volume dependent but in many designs the propulsion plant is fixed by prior development. If there is freedom to tailor the plant to meet operational requirements it is possible to start with a low estimate and iterate up to the required performance.

The question of whether to start the concept on a weight or space basis may seem academic since they must both balance. It is my experience that, because of the close interrelation, there is a self-satisfying process at work which tends to justify the initial weight estimate and may result in an over-large solution. The space-based approach permits a smallest possible first estimate and provides some assurance that the final solution is the smallest compatible with the requirements.

A second point for discussion relates to the structures section. The paper deals with ring-stiffened cylinders, bulkheads and end closures as separate entities but I would like to have seen some discussion of the problems of joining domes to cylinders and the transition junctions between large- and small-diameter hull elements. These can pose computational difficulties and give problems in achieving acceptable stress levels due to the complex strain relationships. My advice to students is to keep the pressure hull simple and avoid complex geometries though it must be accepted that this is not always possible.

R. B. Couch, Member

I am pleased to see this paper by one if not the only U.S. submarine designer of note in the past 20 years or so. Harry is a worthy successor to Admiral Andy McKee, who was the preeminent submarine designer of the past decades.

This paper is a practical exposition of the process involved in producing a modern submarine design. The author is a practical designer and the paper certainly follows this precept.

I cannot criticize this paper in detail since I have been out of the business for some time. However, I would like the author to answer a question. What do you know about foreign submarine design, in particular, Russian? Are they following the methods outlined by the author?

Otherwise, I can only compliment the author on a job well done. I hope he won't quit just yet.

Frank W. Wood, Member

While the author has left the brief discussion of arrangements until the end, I think it is the place where the concept begins. As the author states, "The arrangement is much more difficult in a submarine than in any other vehicle." In a surface ship, the superstructure can be enlarged to accommodate some volume growth after the main hull has been defined. In the submarine, any volume deficiency must be overcome by increase in length after the diameter has been set.

The probable driving force for an entirely new concept design will be a new or novel combat system, and the relative location and arrangement of the sensors, launchers, weapon storage, and fire control equipment will be the most important aspect of the design outside of the machinery spaces. After all, the combat system is the payload of the military submarine, or any other warship. So the process begins with arrangement studies of these areas. For this purpose the method of defining preliminary hull shape suggested by the author is very useful.

I wish the author had been more specific about the discrete hull diameters for best space utilization. In any case, a two-level arrangement will fall in the 16 to 25 ft range, three levels in the 28 to 35 ft brackets and four levels in the 38 to 44 ft area. These allow for intermediate deck heights of 7 to, say, 8½ ft with more height being allowed for the upper and lower levels to provide for the curve of the hull. With large diameters, reserve buoyancy (MBT volume) must be provided to limit normal surface condition draft to somewhere around 30 ft. In a nuclear-powered submarine the final diameter is usually the subject of interchange with the nuclear power authority.

Because of the heavy concentrations of weight of the reactor compartment, it needs to be more or less centered at the LCB/LCG. Any imbalance can be offset to some extent by the judicious location of the battery compartment, if provided.

In a nonnuclear submarine, the storage battery tanks, usually two, must likewise be located so that their combined center is somewhere close to the LCB/LCG. The batteries will probably each consist of 120 cells which may be arranged as 20 rows of six abreast. While there may be some trade-off of dimensions of the cells by discussion with the battery provider, their volume is fixed by the capacity required, and so the breadth and length of each tank can be determined early. Since the batteries are heavy, they belong in the lower level and the cell height and hull diameter will set the height of the first level, allowing for access for servicing.

The hull diameter and machinery arrangement will, of course, set the length of the engine room.

With a nonnuclear single-screw, direct-drive electric plant, the diameter of the main propulsion motor(s) will have a major influence on the diameter of the hull at the after end of the engine room.

Arrangement studies of communication, command, and control equipment should be done using the equipment generated by the mission requirements and the relative location of such spaces can then be allocated, driven to some extent by the periscopes, as noted by the author.

Finally, if a neat volume-weight balance is the goal, the spaces remaining after the trim tanks have been located can be utilized for habitability areas. If more space is needed for this purpose, it will be at the expense of increased length and lead ballast since the density of such spaces is less than one. An alternative would be to increase the weight of the pressure hull to add to the test depth or improve the factor of safety.

C. L. Long, Member

Harry notes that submarine design has been revolutionized by nuclear power, body-of-revolution hull forms, and personal computers. Although I agree that personal computers make our work as designers easier, it is the advent of CAD systems that has revolutionized submarine design. Today with our CAD systems the arrangement of a ship can be developed in a fraction of the time it took us with triangles and tee squares. I was involved in doing fifteen forward end arrangements and balancing each arrangement in less than two months through the use of CAD tools. This brings me to Fig. 1 of the paper.

Figure 1 does not show the order with which one should approach arriving at the ship size, which will be a balanced ship and contain the equipment that will meet the mission requirements. First and foremost, one must have the mission requirements. Secondly, an arrangement must be produced within a hull envelope, which will contain the propulsion system being contemplated, combat system including weapons launching and handling, crew living, the space to incorporate the noise and shock requirements, and shall retain adequate space for ventilation and piping systems. Thirdly, powering calculations must come next to see if the powering assumption made for arrangements purposes is adequate to meet the ship speed requirements for the hull envelope required. Fourth, structural calculations need to be done. As shown in Fig. 3 and Table 1 of the paper, the hull weight can be a large contributor to the Normal Surface Condition (NSC). (The abscissa in Fig. 3 should be labeled either test or collapse depth depending on the basis used in developing the curve.) Weight and ship balance can now be made with the major contributors defined. I prefer to look at the design process, not as a flow diagram as shown in Fig. 1, but as a design spiral such as shown in Fig. 29 herewith. Of course, many additional milestone spokes can be put on the design spiral. Concept design is influenced heavily by past designs or studies.

In the text it is said that the group 4 weights, that is, electronics weights are "a function of the weapons system." I don't believe that is true where "weapons systems" represents the weapons stowage and handling and launching components, and it isn't consistent with Table 1, which notes that the basic electronics is given by the ship requirements or mission.

Lead margin, that is, lead that the ship carries which is not assigned to stability or trimming the ship is very important. This lead margin must take care of:

1. Design margin—lead assigned to cover design weights which are heavier than estimated.
2. Shipbuilder margin—lead assigned to cover shipbuilder and vendor equipment manufactured weights, which are heavier than design weights.

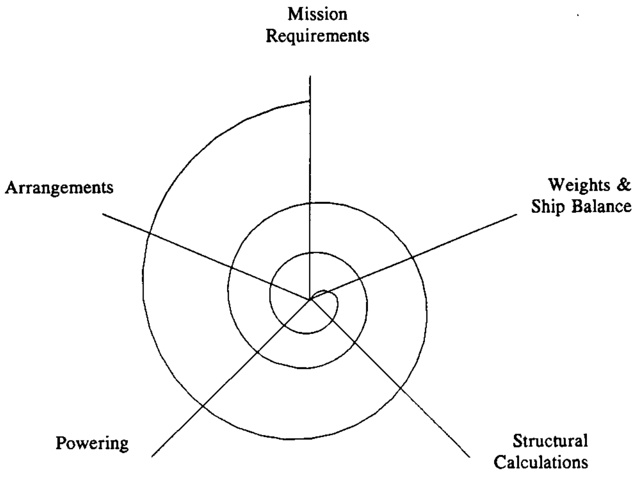

Fig. 29 Design spiral

3. Future growth margin—lead assigned to the ship to cover future ship weight growth.

The author gave the ratio of lead to the A-1 weight as 10–12.5%. This is probably on the high side of what is normally used. It would be interesting to see data that showed the trend of ship's margin with time through the design period. Likewise the trend of ship's margin with time through the life of operating ships.

On page 424, it is said that the offsets can be determined from Figs. 21 and 22 in the Appendix. This should refer to Table 4 and Figs. 22 and 23.

In equation (8b) it could be interpreted that NSC is divided by 64 rather than indicating that the value for NSC assumes that the submarine is floating in 64 # / cu ft water.

On page 426, the denominator of equation (10) for C_f is incorrect. The denominator should be within parentheses and squared, not just (RE-2).

Equation (20) shows $k = A_f/A_s$ where, I believe, it should be $k = A_f/(A_f + A_s)$ where k is defined as the portion of the total load carried by the frame. I believe the range of k will be 0.25 to 0.30 for normal submarine frame shell designs. Figure 30 shows the development of k.

On page 431 it is said that the structure of one frame space can be considered to be made up of two columns, one the shell and the other the frame and the shell. Is the word columns intended or should it be "components"?

In equations (24a) and (24b), shouldn't R be R_S and R_f be R_{NA}? I found that symbols were not consistently used throughout the paper, which made it difficult to read and maybe a little dangerous to use without researching.

I would recommend adding to the sentence "Fatigue is the primary concern" on page 432 the following, "for the factor of safety of 1.5 on yield stress." Fatigue is not a concern with the lobar buckling and general instability factors of safety.

Equation (39) is listed as the buckling stress for a hemispherical end closure. This is the equation for the plate stress due to pressure, not the buckling stress. Equation (40) gives the buckling pressure, P_B, not the stress. Substituting P_B for P in equation (39) gives the buckling stress in the head $\sigma_B = 0.42Et/R_1$. It is recommended that the critical length L provided between equation (39) and (40) be dropped because it isn't used in equation (40) and it has a typo. The ½ power applies to R_1 as well as to t_1.

K	=	Portion of Total Load Carried by a Frame
P	=	External Pressure
R_o	=	Shell Radius
L_f	=	Frame Spacing
A_f	=	Area of Frame
σ_f	=	Stress in Frame
A_s	=	Area of Shell
σ_s	=	Stress in Shell

Total Load on Shell & Frame = $P\ R_o\ L_f$

Frame Load = $K\ P\ R_o\ L_f$

$$\sigma_f = \frac{K\ P\ R_o\ L_f}{A_f}\ ;\ \text{and}$$

$$\sigma_s = (1 - K)\ \frac{P\ R_o\ L_f}{A_s}$$

let $\sigma_f = \sigma_s$

$$\frac{K\ P\ R_o\ L_f}{A_f} = (1 - K)\ \frac{P\ R_o\ L_f}{A_s}$$

$$K\ A_s = A_f - K\ A_f$$

$$K = \frac{A_f}{A_f + A_s}$$

Fig. 30 Development of equation (20)

It is worthy to note that higher-strength steels allow higher design stress. As shown in equation (41), higher radial hull deflections will occur as pressure hull stress is increased. Therefore, greater care in locating equipment in the ship is necessary so that the larger radial and longitudinal deflections can be accommodated. In a submarine where normally every cubic foot is needed, this may not be a trivial matter.

I agree totally with Harry's conclusion that designing a submarine is a complex undertaking and that the design process and knowledge takes years to develop. I feel that the real value of this paper is its usefulness in providing an understanding of the unique relationships that exist in submarine design. Our thanks to the author for presenting this paper.

Russell W. Brown, Member

This is the first submarine paper that has been presented by SNAME in over 30 years. We are grateful for the effort to prepare this paper.

A more accurate title for the paper would have been "Naval Architectural Elements of Submarine Design."

There are four phases in submarine design: concept, preliminary, contract, and final. Concept designs are rough cuts of designs with various features to provide choices for selection of a design to develop. Preliminary design develops the desired concept to the degree required to define the ship relative to speed, power displacement, weights arrangement of all spaces and structure. Contract design develops the preliminary design to the degree that

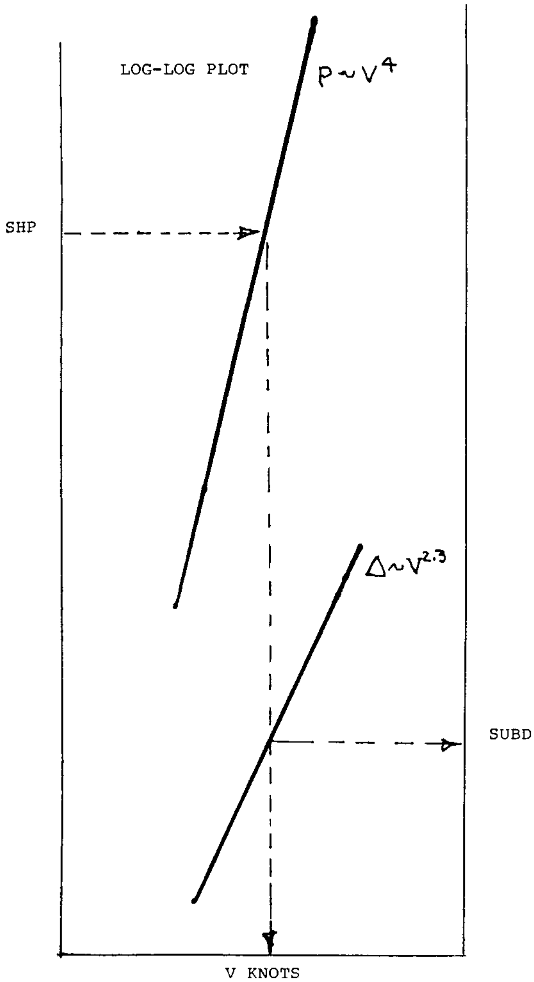

Fig. 31 Speed-power-displacement curves

$$\text{Drag} = \frac{1}{2}\rho A V^2 C_f \qquad C_f = \frac{0.0735}{(Re)^{0.2}} \qquad Re = \frac{DV\rho}{\mu}$$

Power, $P \cong \text{Drag} \cong V$

If Δ = Displacement, $A \approx \Delta^{2/3}$; $D \approx \Delta^{1/3}$

$$\text{Then } P \cong \frac{\Delta^{2/3} V^2 V}{(\Delta^{1/3} V)^{0.2}}$$

$W \approx \Delta \approx P^{1/2}$

$$\text{Then } P \cong (P^{0.5})^{0.6} V^{2.8} \cong P^{.3} V^{2.8} \text{ or } P^{0.7} \cong V^{2.8} \text{ or } P \cong V^4$$

$$\text{and } \Delta^{0.6} \cong \frac{P}{V^{2.8}} \cong \frac{V^4}{V^{2.8}} \cong V^{1.2} \text{ or } \Delta \cong V^2$$

Russ: This must be why our curves plot the way they do!

I understand that the curves actually indicate that:

$P \cong \Delta^{0.52} V^{2.8}$

$\Delta \cong P^{0.58}$

$\Delta \cong V^{2.3}$

$P \cong V^4$

Fig. 32 Derivation of speed-power-displacement relationship for submarines by John S. Leonard (1979).

bids for detail design and construction can be obtained. Detail design is used to construct the ship.

This paper covers naval architectural features of concept and preliminary design.

In this paper, ship displacement is determined by developing all the weights for each of the seven groups. A quick alternate way to find the ship displacement, speed and power is by using the curves of speed, power and displacement shown in Fig. 31 herewith. The curves are applicable to approximate geosyms of the same standards of design. The curves have been verified empirically and mathematically by the John S. Leonard (1979) derivation (Fig. 32).

The ship envelope can be developed from the selected diameter and fore and aft body geometries. The midbody length is adjusted to obtain the displacement found from the curves.

The volumes of pressure hull compartments are selected by the relationship $V \sim P^{1/2}$ and by comparison with past designs. The fixed water ballast is arranged fore and aft of the pressure hull with space for sonar, steering and diving equipment, and main shaft.

Weights are estimated per the paper. The amount of lead is the difference between all the weights and the displacement. If lead is not within acceptable limits, hull size can be adjusted.

Drag factors for hull and appendage resistance as shown on the Mandel-Arentzen curve can be obtained from *PNA* and the ehp can be developed. Using an appropriate PC the shp can be determined. Any difference between the desired and calculated shp can be corrected by adjusting ship speed.

The propeller design approach in the paper is more involved than needed for concept or preliminary design. For SSNs, the propeller diameter can be assumed to be half of the ship diameter with a pitch ratio of one. Shaft speed can be determined by assuming slip of 0.1, wake fraction of 0.3 and by using ship speed. This is needed for machinery size estimating. Weights and moments and polygon can be determined per the paper.

Structure can be developed from the 1967 edition of *PNA*, first printing. Also, structural information can be found in *Structural Design of Warships*, by Hovgaard, published in 1940 by the Naval Institute. The polygon is explained in a 1947 New England Section paper by A. I. McKee.

In conclusion, this paper provided useful information for submarine design, especially in the weight area.

E. Eugene Allmendinger, Member

Thanks for a very fine paper, Captain Jackson. It is high time this subject surfaced again for Society deliberations.

I regret that I have not had time to review this paper in any great detail but will offer a few thoughts. I would prefer to see the "Design Spiral" used rather than the Feasibility Study Flow Chart of Fig. 1. In my view, the iterative nature of the concept (as well as the preliminary) design process is more clearly illustrated by the spiral than by the flow chart. If the chart in Fig. 1 is used, I would think that there would be a "feedback" line from the important design consideration "Arrangement" box. The chart seems to say that the arrangements have to fit in a given length and volume, take it or leave it.

Looking at Fig. 23 in the Appendix, the abscissa scale is inverted for the curves as drawn or vice versa. For $Xa/La = 0.0$, $Ya/(D/2)$ should be 1.0000 and for $Xa/La = 1.0$, $Ya/(D/2)$ should be 0.0000.

Authors' Closure

Several discussers stated that they preferred to start the design spiral with arrangements. There is no question that the arrangements are the crux of the concept of preliminary design and are of utmost importance. It is the area where the skill of the designer is put to the extreme test. The paper presents only one approach. Mr. Long presents a spiral of design progress. All phases must be investigated along the spiral, but it can be started at any point on the outer ring. A designer must have some concept in mind at the start. He will build on this concept and refine it along the way. The databanks available to the designer will have a large influence on the development of the design. My databank is based on the weight groups listed in Table 1. It works because most of the data are derived from submarines that have been successfully completed and there is an acceptable relationship between the weight and volume. Equation (1) could also be developed for volumes and be equally satisfactory. As a matter of fact, I have done this for AIP submarines where the variable load is usually not very heavy and requires excessive volumes.

Equation (1) and Table 1 are useful only to determine the first approxatimation of the principal dimensions that are required to begin an arrangement study. They also can be used effectively to answer the "what if" questions that arise when considering adding or removal of equipment.

Table 2 can be developed only after some preliminary arrangements are sufficiently completed to establish the LCG and VCG of the weight groups.

An estimate of the principal dimensions must be made in order to calculate the power required to make the speed listed in the requirements. This will either confirm or alter the weight and size estimates used in creating Table 1.

Mr. Wood's comments are well taken and are covered in detail in Mr. Fast's comments. His other comments are general in nature and add greatly to the value of the paper. I concur with his expressed philosophy.

Mr. Fast presents an excellent description of the problems of selecting the correct pressure hull diameter. His presentation is a well thought out and lucid summary of the importance of selecting the proper diameter. One of the difficulties of utilization of the spaces between the frames is clearly shown in his Fig. 25.

Mr. Long's comments are a clear indication of the many changes that have taken place in the past couple of decades. CAD design capabilities have indeed changed the design process and made it much better and quicker than at any time in the past. He did not mention it directly, but indirectly he showed that the principal designer does the detail work and eliminates the interchange between designer and the draftsman. A very important consideration. When I commented on the importance of the computer, I had in mind CAD, which can be considered as an extension of the computer.

The flow direction of the design process in Fig. 1 is vertically downward on the right-hand side. It starts with the requirements. The spiral of design has been presented in ever so many ways, and I was searching for a different approach. Both presentations indicate the same thing. The order may be changed without affecting the final outcome. The only difference in the approaches is that I try to pin down the principal dimensions and define the envelope prior to making the arrangement studies. I had prepared a section on arrangements that was similar to Mr. Long's, but removed it in order to stay within the confines of the specified length of the paper. Arentzen covered the subject very well in his paper (Ref 1). The amount of lead to be carried in order to provide a margin for the items Mr. Long describes is a very complex determination. It is compounded by the fact that the weight and volume estimates of the various components have hidden margins in them as well. My ratios are based on past experiences. The amount of the margins to be included is a function of the correctness of the weight and volume databanks. If there is good confidence in the data, the margins can be reduced. Where there are long periods between design programs, the confidence is reduced and the margins tend to be larger. My experience with margins in new design is that the trend is always down and must be rigidly controlled. One indication of the end of the useful life is the point where the margins are all used up. My records indicate that one submarine had 300 tons of margin lead upon completion and only 6 tons at decommission. The low remaining margin was a factor in eliminating her from the active fleet.

Equation (8b) is in error, and will be corrected in the final printing.

The square in the denominator is in my copy of the paper. The squared denominator is correct.

Regarding equation (20), Mr. Long's formula is correct. However, my databank is based on the ratio of the area of the frame to the area of the shell and the ratios are in the range indicated. Using my ratios in his equation, the results are the same.

Regarding the comments on Dr. Kendrick's equation (24a), I used the term "columns" because that is what Dr. Kendrick called them. In retrospect, "components" is probably a more descriptive term.

Regarding equations (24a) and (24b), technically the terms should be as suggested; however, the equations are as described by Dr. Kendrick and so were not changed. The final results are not much different regardless of which is used and are within the accuracy of the formulas.

There is much debate about the effect of fatigue in structure that supposedly is always under compressive load. In his May 1952 *ASNE Journal* paper, Heller pointed out that there are considerable tensile stresses in a submarine hull as it changes depth, and this results in a reduction of strength over time. Buckling failures are often considered to be instantaneous; however, model tests have indicated that there is considerable deformation prior to the complete devastating failure. As a structure is eccentrically loaded, a bending stress is created with both compression and tension components. The tension components are of concern. I think that the statements in the paper that fatigue is a concern in the buckling mode are correct. To ignore it is to invite problems in the later stages of the submarine's operating life.

In reply to Dr. Abels, the concepts presented in the paper are applicable to all sizes, even as small as torpedoes. They are also applicable to hulls with sections other than circular if they are slightly modified. Regarding noise, the statement "The quieter, the better" is appropriate. The real question is "How much can be afforded?" It is quite possible to incorporate noise reduction beyond the point of diminishing returns.

Tonnage is always a concern, and large efforts should always be in the direction of keeping the submarine as small as possible. Frequently, the requirements specify a range of displacements to be considered which limits the designers' options for investigation.

Land's paper in SNAME *Transactions* of 1919 outlines the pros and cons of single and double hulls of submarines. He points out that the selection of one or the other depends on many factors. But most important is the mission of the submarine. In general, fossil fuels favor double hulls and nuclear power favors single hulls. Structural considerations are also of utmost importance.

I think that there are only five points that need to be considered in preparing the polygon. They are heavy, light, heavy forward, heavy aft and the normal condition which results from the initial weight summary in Table 2.

The best way to take advantage of higher strength steels in submarines is to go deeper. This is due to the buckling failure modes which depend on moments of inertia, which are independent of steel strength. This is shown clearly in Fig. 3.

Some designers design their structures to fail in the "one-horse shay" modes. That is, all failure modes occur at the same time. The buckling modes can be triggered by imperfections in construction and welding which are very difficult to predetermine. Also, submarine hulls often suffer minor damages whose impact on strength is difficult to assess. Therefore, it seems prudent to include an extra factor of safety for these modes of failure. As it turns out, it does not cost much in weight because it reduces the bending stress in the frames as indicated in equations (36) and (38).

Out-of-roundness, frame tilt, and out-of-plane frames are all considered and limitations are included in the specifications.

Snorkel considerations were omitted from the paper due to the limitations on length. Much data are available in the open press. For air-dependent submarines, this is a major factor in the design.

Replying to Dr. Burcher's comments, the path starting with the arrangements has been discussed previously. The design of the structures should indeed be kept as simple as possible. I concur the transitions are of great concern and must be investigated in great detail. I briefly mention the end closures that are made up of combinations of sphere segments, cones and ellipsoids. These not simple but acceptable designs have been worked out.

Mr. Brown's comments discuss the source of much of the information in the paper and its references, some of which are included in the paper. He substantiates the unclassified nature of all of the material in the paper. In a private note he states that he hopes that this paper will open the door to many more papers involving submarines. That is one of the objectives of the paper and I hope that younger authors will accept the challenge to present many more. The curves presented by Mr. Brown are very useful for estimating the size and power of a proposed submarine; however, the results must be considered as approximations only and must have hull shapes similar to the submarines from which the data were developed. It would be useful to have similar procedures that are used by others.

I sincerely appreciate the comments of Professor Couch and thank him very much. There is a great deal of information about foreign submarines available. I have textbooks on submarine design from at least five different countries. Because they all have to live with the laws of physics, there is a great deal of similarity in the designs from the different nations. As an aside, one can see the influence of the *Albacore* and *Barbell* in most of the designs.

I wish to thank the members of the Papers Committee and the SNAME and M.I.T. staffs for their help. Particularly, I want to thank all of the discussers for their interest and contributions. They are the ones who really make the paper worthwhile.