Appendix A

Calculation and Pressure Hulls under External Pressure

A. Introduction

A method of calculation for designing the pressure hulls of submersibles is described below, which can be used for the three loading conditions:

– nominal diving pressure p_N

– test pressure p_P

– collapse pressure p_Z

to investigate the stresses in the pressure hull and the corresponding states of stability:

– asymmetric buckling between stiffeners (axial buckling)

– symmetric buckling between stiffeners (circular buckling)

– general instability of pressure hull design

– tripping of stiffening rings

– buckling of dished ends.

The method of calculation presented takes limited account of fabrication relevant deviations from the ideal shape of the shell (out-of-roundness). Methods of verifying the roundness of hull shells are also described.

Conical shells are calculated in sections, each of which is treated as a cylindrical shell.

Overall collapse of the design is regarded as buckling of the hull structure between bulkheads or dished ends.

With regards to the stresses in the pressure hull the permissible values are those stated in Chapter 2 – Submersibles, Section 4, E.

For the states of stability described, proof is required of sufficient safety in respect of the particular form of damage concerned.

When using the method of calculation it is to be remembered that both elastic and elastic-plastic behaviour can occur in the materials of the shell structure.

It is generally the case that:

– at nominal diving pressure, the stress is within the purely elastic range of the material;

– at test pressure, the stress may lie at the commencement of the elastic-plastic range of the material;

  However, calculations relating to the permissible stress being exceeded can be based on the assumption that the behaviour of the material is elastic.

– at the collapse pressure, the stress may lie in the elastic or the elastic-plastic range of the material.

In the elastic-plastic range, use of the method requires the determination of values by a process of iteration. The modulus of elasticity E and the Poisson's ratio ν shall be substituted by the values E' and ν' according to G. The corresponding iteration techniques are shown as program sequences at the end of the Appendix (Figs. 9 - 11).

When calculating a pressure hull use is to be made of design data corresponding to the proposed service conditions of the submersible in accordance with Chapter 2 – Submersibles, Section 4, E. of these Rules.

It is advisable to use a programmable computer for performing the calculations.

Pressure hulls subjected to internal overpressure are, in addition to be designed in accordance with GL Rules for the Classification and Construction, Part 1 – Seagoing Ships, Chapter 2 – Machinery Installations, Section 8.

B. Stiffened and Unstiffened Cylindrical Shells

1. General

For the loading conditions mentioned in A. Cylindrical shells are to be checked for excess stresses and asymmetric and symmetric buckling.

The method of calculation presented below is for stiffened cylindrical shells. In the case of unstiffened cylindrical shells with dished ends, the calculations are performed in a similar manner, the cross-sectional area of the ring stiffener being $A = A_1 = 0$ and the spacing between stiffeners being defined by the ends. Where the spacing between stiffeners is defined by dished ends 40 % of the depth H of each dished end is to be added to the cylindrical length (see Fig. 1). The "general stability" state is considered in C.3.3.

for the calculation of buckling in the elastic-plastic range the modulus of elasticity E and the Poisson's ν is determined by applying formulae (65) - (68) and by means of the stress σ_i formula (1) in the centre of the section and the centre of the plate.

The calculations allow for an out-of-roundness of the shell of maximum u = 0,005. If larger tolerances are planned, or if the method of measurement described in H.1 results in greater out-of-roundness values, then the permissible pressure is to be checked in accordance with H.2.

2. Stresses in the cylindrical shell

The stress intensity (at the centre of the plate at mid-bay position between ring stiffeners) is determined by applying formulae (1) - (14).

In formulae (2a) to (2d) the bending component is expressed by the plus sign on top for the outside of the cylindrical shell and by the minus sign below for the inside. The stresses in the centre of the plate are determined by omitting of the expressions after the plus/minus signs.

$$\sigma_i = \sqrt{\sigma_x{}^2 + \sigma_\varphi{}^2 - \sigma_x \cdot \sigma_\varphi} \qquad (1)$$

$$\sigma_o = -\frac{p \cdot R}{s} \qquad (2)$$

In the centre of the section the following applies:

$$\sigma_x = \sigma_o \left(\frac{1}{2} \pm C_{10} \cdot C_{11} \cdot F_4 \right) \qquad (2a)$$

$$\sigma_\varphi = \sigma_o \left(1 - C_{10} \cdot F_2 \pm \nu \cdot C_{10} \cdot C_{11} \cdot F_4 \right) \qquad (2b)$$

In the area of stiffening the following applies:

$$\sigma_x = \sigma_o \left(\frac{1}{2} \pm C_{10} \cdot C_{11} \cdot F_3 \right) \qquad (2c)$$

$$\sigma_\varphi = \sigma_o \left(1 - C_{10} \pm \nu \cdot C_{10} \cdot C_{11} \cdot F_3 \right) \qquad (2d)$$

$$F_1 = \frac{4}{C_5} \left[\frac{\cosh^2 C_8 - \cos^2 C_9}{\frac{\cosh C_8 \cdot \sinh C_8}{C_6} + \frac{\cos C_9 \cdot \sin C_9}{C_7}} \right] \qquad (3a)$$

$$F_2 = \left[\frac{\frac{\cosh C_8 \cdot \sin C_9}{C_7} + \frac{\sinh C_8 \cdot \cos C_9}{C_6}}{\frac{\cosh C_8 \cdot \sinh C_8}{C_6} + \frac{\cos C_9 \cdot \sin C_9}{C_7}} \right] \qquad (3b)$$

$$F_3 = \sqrt{\frac{3}{\left(1-\nu^2\right)}} \left[\frac{\frac{-\cosh C_8 \cdot \sinh C_8}{C_6} + \frac{\cos C_9 \cdot \sin C_9}{C_7}}{\frac{\cosh C_8 \cdot \sinh C_8}{C_6} + \frac{\cos C_9 \cdot \sin C_9}{C_7}} \right] \qquad (3c)$$

$$F_4 = \sqrt{\frac{3}{\left(1-\nu^2\right)}} \left[\frac{\frac{\cosh C_8 \cdot \sin C_9}{C_7} - \frac{\sinh C_8 \cdot \cos C_9}{C_6}}{\frac{\cosh C_8 \cdot \sinh C_8}{C_6} + \frac{\cos C_9 \cdot \sin C_9}{C_7}} \right] \qquad (3d)$$

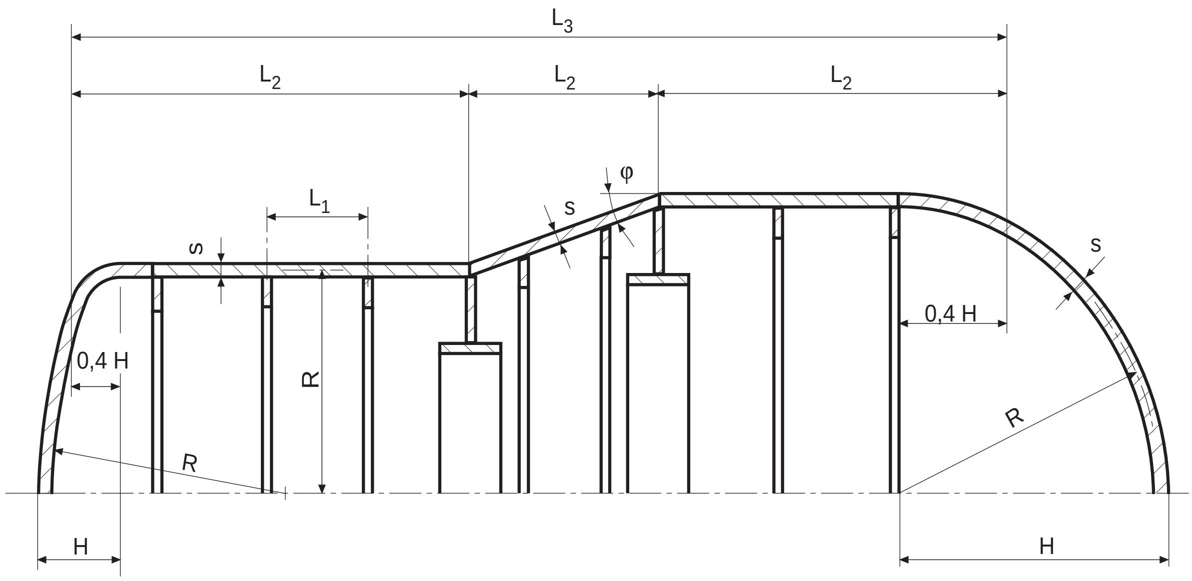

Fig. 1 Spacing between stiffeners in pressure vessel subject to external pressure

$$A = A_1 \cdot \frac{R^2}{R_o^{\,2}} \quad (4)$$

$$C_5 = \alpha \cdot L_1 \quad (5)$$

$$C_6 = \frac{1}{2}\sqrt{1-G} \quad (6)$$

$$C_7 = \frac{1}{2}\sqrt{1+G} \quad (7)$$

$$C_8 = C_5 \cdot C_6 \quad (8)$$

$$C_9 = C_5 \cdot C_7 \quad (9)$$

$$C_{10} = \frac{\left(1-\frac{\nu}{2}\right)\cdot\frac{A}{s\cdot L_1}}{\frac{A}{s\cdot L_1}+\frac{b}{L_1}+\left(1-\frac{b}{L_1}\right)F_1} \quad (10)$$

$$C_{11} = \sqrt{\frac{0{,}91}{1-\nu^2}} \quad (11)$$

$$p^* = \frac{2\cdot s^2\cdot E}{R^2\cdot\sqrt{3\cdot\left(1-\nu^2\right)}} \quad (12)$$

$$G = \frac{p}{p^*} \quad (13)$$

$$\alpha = \sqrt[4]{\frac{3\cdot\left(1-\nu^2\right)}{s^2\cdot R^2}} \quad (14)$$

$$K_o = \frac{\sigma_\varphi}{\sigma_x} \quad (15)$$

3. Provision against excess stresses

The stress intensity for the three loading conditions is obtained from formula (1). Sufficient safety against exceeding the permissible stress is provided if the conditions (16a, b, c) are met. In formulae (2a) to (2d) where load $p = p_Z$ the binding component can be disregarded.

$$k = \geq \sigma_i \cdot S \quad (\text{where } p = p_N) \quad (16a)$$

$$k = \geq \sigma_i \cdot S' \quad (\text{where } p = p_P) \quad (16b)$$

$$k = \geq \sigma_i \quad (\text{where } p = p_Z) \quad (16c)$$

The program sequence (see Fig. 9) shows the method of calculation used to obtain, for the loading conditions collapse pressure, the stress intensity and the maximum permissible pressures for the permissible stresses.

4. Asymmetric buckling

The buckling pressure p_n is calculated with formulae (17) - (19) for the integer value $n \geq 2$ corresponding to the lowest value of p_n. The relevant stresses of the centre of the plate are determined in accordance with 2.

The program sequence (see Fig. 9) shows the method of calculation used to determine the buckling pressure p_n in the elastic-plastic range.

$$p_n = \frac{E\cdot s\cdot\beta_{n1}}{R} \quad (17)$$

$$\beta_{n1} = \frac{\left[\left(\frac{n^2}{\lambda_1^{\,2}}+1\right)^{-2}+\frac{s^2\cdot\left(n^2-1+\lambda_1^{\,2}\right)^2}{12\cdot R^2\cdot\left(1-\nu^2\right)}\right]}{\left(n^2-1+0{,}5\cdot\lambda_1^{\,2}\right)} \quad (18)$$

$$\lambda_1 = \frac{\pi\cdot R}{L_1} \quad (19)$$

5. Provision against asymmetric buckling

The buckling pressure for the three loading conditions is obtained from formula (17). Sufficient safety against asymmetric buckling is provided if the conditions (20a, b, c) are met.

$$p_n \geq p_N \cdot S_k \quad (20a)$$

(for the nominal diving pressure load condition)

$$p_n \geq p_P \cdot S_k{}' = p_N \cdot S_1 \cdot S_k{}' \quad (20b)$$

(for the test diving pressure load condition)

$$p_n \geq p_Z = p_N \cdot S_2 \quad (20c)$$

(for the collapse pressure load condition)

6. Symmetric buckling

The buckling pressure p_m is calculated with formulae (21) - (33) and (15) for the lowest integer value of m at which conditions (33) is met. The values E_s and E_t are determined in accordance with G. The relevant stresses of the centre of the plate are calculated in accordance with 2. In the elastic range $E_s = E_t = E$ and $\nu' = \nu$.

The program sequence (see Fig. 10) shows the method of calculation used to determined the buckling pressure p_m in the elastic-plastic range.

$$p_m = p^{**}\cdot C_o\cdot\left[\left(\frac{\alpha_1\cdot L_1}{\pi\cdot m}\right)^2+\frac{1}{4}\cdot\left(\frac{\pi\cdot m}{\alpha_1\cdot L_1}\right)^2\right] \quad (21)$$

$$p^{**} = \frac{2\cdot s^2\cdot E_s}{R^2\cdot\sqrt{3\cdot\left(1-\nu'^2\right)}} \quad (22)$$

$$C_o = \frac{\sqrt{C_1 \cdot C_2 - \nu'^2 \cdot C_3^2}}{1 - \nu'^2} \quad (23)$$

$$C_1 = 1 - \frac{H_2^2 \cdot H_4}{H_1} \quad (24)$$

$$C_2 = 1 - \frac{H_3^2 \cdot H_4}{H_1} \quad (25)$$

$$C3 = 1 + \frac{H_2 \cdot H_3 \cdot H_4}{\nu' \cdot H_1} \quad (26)$$

$$H_1 = 1 + H_4 \cdot \left[H_2^2 - 3 \cdot (1 - \nu'2)\right] \quad (27)$$

$$H_2 = (2 - \nu') - (1 - 2\nu') K_o \quad (28)$$

$$H_3 = (1 - 2\nu') - (2 - \nu') K_o \quad (29)$$

$$H_4 = \frac{1 - \frac{E_t}{E_s}}{4 \cdot (1 - \nu'^2) \cdot K_1} \quad (30)$$

$$K_1 = 1 - K_o + K_o^2 \quad (31)$$

$$\alpha_1 = \sqrt[4]{\frac{3 \cdot \left[\frac{C_2}{C_1} - \left(\nu' \cdot \frac{C_3}{C_1}\right)^2\right]}{s^2 \cdot R^2}} \quad (32)$$

$$\frac{\alpha_1 \cdot L_1}{\pi} \leq \sqrt{\frac{m}{2}(m+1)} \quad (33)$$

7. Provisions against symmetric buckling

The buckling pressure p_m for the collapse pressure condition is obtained from formula (21). Sufficient safety against symmetric buckling is provided if the conditions (34a, b, c) are met.

$$p_m \geq p_N \cdot S_k \quad (34a)$$

$$p_m \geq p_P \cdot S_k' = p_N \cdot S_1 \cdot S_k' \quad (34b)$$

$$p_m \geq p_Z = p_N \cdot S_2 \quad (34c)$$

C. Ring Stiffeners

1. General

It is the purpose of ring stiffeners to reduce the buckling length of cylindrical shells. A distinction is made between "heavy" and "light" ring stiffeners. "Heavy" ring stiffeners are stiffeners which are able to reduce the significant mathematical length of the pressure hull as this relates to the failure described in 2.3. The dimensions of "heavy" stiffeners are not to be smaller than the "light" stiffeners.

For a terminal section, the length to be used is that between the end and the stiffener. (In the case of dished ends, the buckling length is to take account of the instruction in B.1 and Fig 1.)

For the loading conditions mentioned in A., stiffeners are to be designed for safety against excess stresses, buckling and tripping. Unreinforced cut-outs in the girth or web are to be considered for calculation.

2. "Light" stiffeners

2.1 Stresses in "light" stiffeners

The stresses are calculated using formulae (4), (14), (35 - (37) and the values of p_{n1} and n according to 2.3. If n = 2 determined also n = 3 has to be calculated. In formula (37), $L = L_1$. Where the distances L_1 to the two adjoining stiffeners are unequal, the calculation shall make use of the arithmetic mean value of both distances. In the elastic-plastic range the values E and ν are replaced by E' and ν' respectively. The elasticity modulus E' and the Poisson's ratio ν' are calculated in accordance with G. In relation to stress σ_f.

The program sequence (see Fig. 11) shows the method of calculation in the elastic-plastic range.

$$\sigma_f = -\frac{p \cdot R^2 \cdot \left(1 - \frac{\nu}{2}\right)}{R_1 \cdot \left[s + \frac{A}{b + \frac{2 \cdot N}{\alpha}}\right]} \quad (35)$$

$$\sigma_{fb} = \pm \frac{p \cdot (n^2 - 1) \cdot E \cdot e_2 \cdot U}{(p_n - p) \cdot R_o^2} \quad (36)$$

$$N = \frac{\cosh(\alpha \cdot L) - \cos(\alpha \cdot L)}{\sinh(\alpha \cdot L) + \sin(\alpha \cdot L)} \quad (37a)$$

$$N = 1 \text{ for } \alpha \cdot L > 5,5 \quad (37b)$$

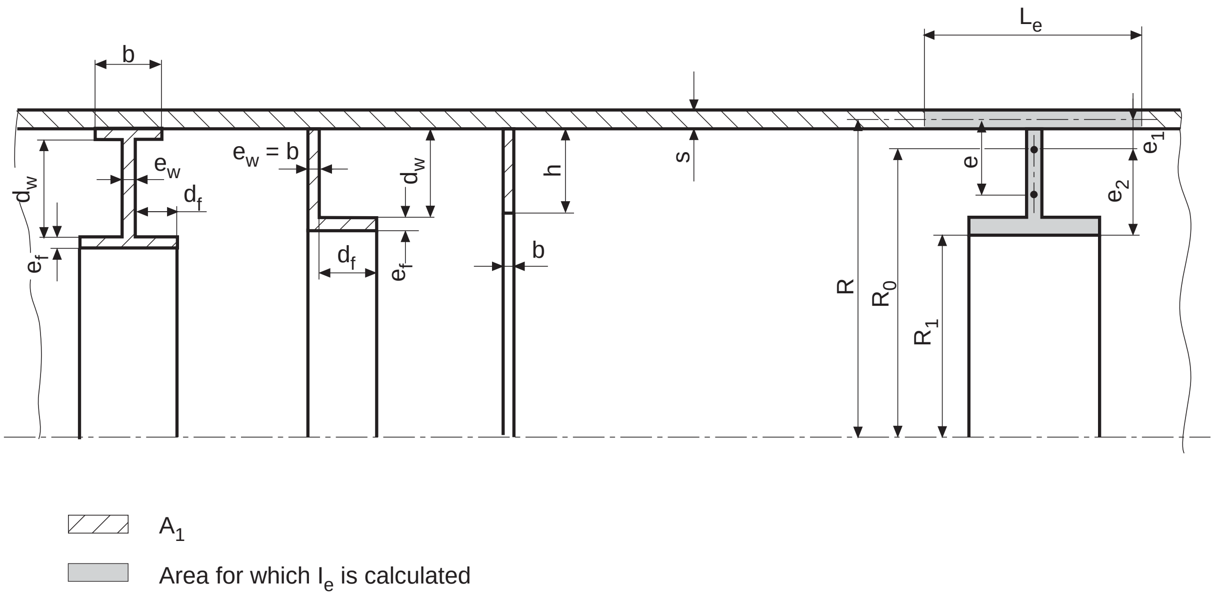

Fig. 2 Stiffeners

2.2 Provision against excess stresses

For the three loading conditions, formulae (35) and (36) give the stresses σ_f and σ_{fb}, the absolute values of which are related to the yield strength k in conditions (38a, b, c).

$$k \geq |\sigma_f| \cdot S + |\sigma_{fb}| \cdot S_k \quad (\text{for } p = p_N) \tag{38a}$$

$$k \geq |\sigma_f| \cdot S' + |\sigma_{fb}| \cdot S_k' \quad (\text{for } p = p_P) \tag{38b}$$

$$k \geq |\sigma_f| + |\sigma_{fb}| \quad (\text{for } p = p_Z) \tag{38c}$$

2.3 Buckling

The "light" stiffeners are to be calculated using formulae (39) - (45) for the integer $n \geq 2$ which produces the lowest value of p_{n1}. In formula (41) $L = L_2$, and, in the absence of "heavy" stiffeners, $L = L_3$. In the elastic-plastic range, E' according to G. is to be substituted for E in formulae (39) and (42). The necessary stress calculation is performed in accordance with 2.1.

$$p_o = \frac{E \cdot s \cdot \beta_{n2}}{R} \tag{39}$$

$$\beta_{n2} = \frac{\lambda_2^{\,4}}{\left(n^2 - 1 + 0,5 \cdot \lambda_2^{\,2}\right) \cdot \left(n^2 + \lambda_2^{\,2}\right)^2} \tag{40}$$

$$\lambda_2 = \frac{\pi \cdot R}{L} \tag{41}$$

$$p_1 = \frac{\left(n^2 - 1\right) \cdot E \cdot I_e}{R_o^{\,3} \cdot L_1} \tag{42}$$

$$p_{n1} = p_o + p_1 \tag{43}$$

$$I_e = \frac{A_1 \cdot e^2}{1 + \frac{A_1}{L_e \cdot s}} + I_1 + \frac{L_e \cdot s^3}{12} \tag{44}$$

$$L_e = \sqrt{2 \cdot R \cdot s} + b \tag{45a}$$

In addition with light stiffeners

$$L_e \leq L_1 \tag{45b}$$

2.4 Provision against buckling

The calculation of the buckling pressure p_{n1} for the three loading conditions is performed in accordance with 2.3. Sufficient safety against buckling is provided if the conditions (46a, b, c) are met.

$$p_{n1} \geq p_N \cdot S_k \tag{46a}$$

(for the nominal diving pressure load condition)

$$p_{n1} \geq p_P \cdot S_k' = p_N \cdot S_1 \cdot S_k' \tag{46b}$$

(for the test diving pressure load condition)

$$p_{n1} \geq p_Z = p_N \cdot S_2 \tag{46c}$$

(for the collapse pressure load condition)

3. "Heavy" stiffeners

3.1 Stresses in "heavy" stiffeners

The stresses are calculated using formulae (35) - (37) and the values p_g and n according to 3.3. In formulae (37) and (41) $L = L_2$. If the distances L_2 to the two adjoining stiffeners (or ends) are unequal, the calculation shall make use of the arithmetic mean value of both distances. In the elastic-plastic range the values E and ν are replaced by E' and ν'. The elasticity modulus E' and the Poisson's ratio ν' are calculated in accordance with G. in the relation to the stress σ_f.

3.2 Provision against excess stresses

For the three loading conditions, formulae (35) and (36) give the stresses σ_f and σ_{fb}, the absolute values of which are related to the yield strength k in conditions (38a, b, c).

3.3 Buckling (general stability)

Using formulae (39) - (42) and (47) - (49), the overall stability of the design is to be calculated for the integer $n \geq 2$ at which the buckling pressure p_g attains its lowest value. The calculation factor C_4 in formula (47) becomes $C_4 = -4$ for internal stiffeners and $C_4 = n^2$ for external stiffeners. Where only one "heavy" stiffener is located midway between two bulkheads, the total buckling pressure p_g formula (49) can be increased by a membrane stress element p_o in accordance with formulae (39) - (41) where $L = L_3$. Where there are no "heavy" stiffeners, the buckling pressure p_g is obtained from formula (43):

$p_g = p_{n1}$

$$p_2 = \frac{(n^2 - 1) \cdot E \cdot I_e}{R_o^2 \, (R + e_1 \cdot C_4) \cdot L_2} \tag{47}$$

$$p_{n2} = \frac{p_o \cdot p_2}{p_o + p_2} \tag{48}$$

$$p_g = p_1 + p_{n2} \tag{49}$$

3.4 Provision against buckling

The calculation of the total buckling pressure p_g for the three loading conditions is performed in accordance with 3.3. Sufficient safety against buckling is provided if the conditions (50a, b, c) are met.

$$p_g \geq p_N \cdot S_k \tag{50a}$$

(for the nominal diving pressure load condition)

$$p_g \geq p_P \cdot S_k' = p_N \cdot S_1 \cdot S_k' \tag{50b}$$

(for the test diving pressure load condition)

$$p_g \geq p_Z = p_N \cdot S_2 \tag{50c}$$

(for the collapse pressure load condition)

4. Tripping of ring stiffeners

4.1 Tripping pressure and general conditions

The tripping pressure p_k of flat bar stiffeners is to be calculated using formulae (4), (14), (37) and (51) and Fig. 3 or 4. The value of n is to be that used in 2.3 or 3.3 for calculations in the elastic-plastic range, E and ν in the aforementioned formulae are to be replaced by E' and ν' in accordance with G. The necessary stress calculation is performed in accordance with 2.1 or 3.1. The maximum allowable value of $(k_1/E) \cdot (h/b)^2$ is 1,14 in each case.

Calculation of the tripping pressure using the formulae referred to above necessitates maintaining the tolerances stated in I.

$$p_k = \frac{k_1 \cdot R_1}{R^2 \left(1 - \frac{\nu}{2}\right)} \cdot \left(s + \frac{A}{b + \frac{2N}{\alpha}} \right) \tag{51}$$

The tripping pressure p_k of L-, T- and I-section ring stiffeners can be calculated by the method described in [3] or [5] (see L. or 4.2).

4.2 Resistance of tripping

For flat bar stiffeners, the tripping pressure p_k for the three loading conditions is obtained from formula (51). Sufficient resistance to tripping is provided if the conditions (52a, b, c) are met.

$$p_{k1} \geq p_N \cdot S_k \tag{52a}$$

(for the nominal diving pressure load condition)

$$p_{k1} \geq p_P \cdot S_k' = p_N \cdot S_1 \cdot S_k' \tag{52b}$$

(for the test diving pressure load condition)

$$p_{k1} \geq p_Z = p_N \cdot S_2 \tag{52c}$$

(for the collapse pressure load condition)

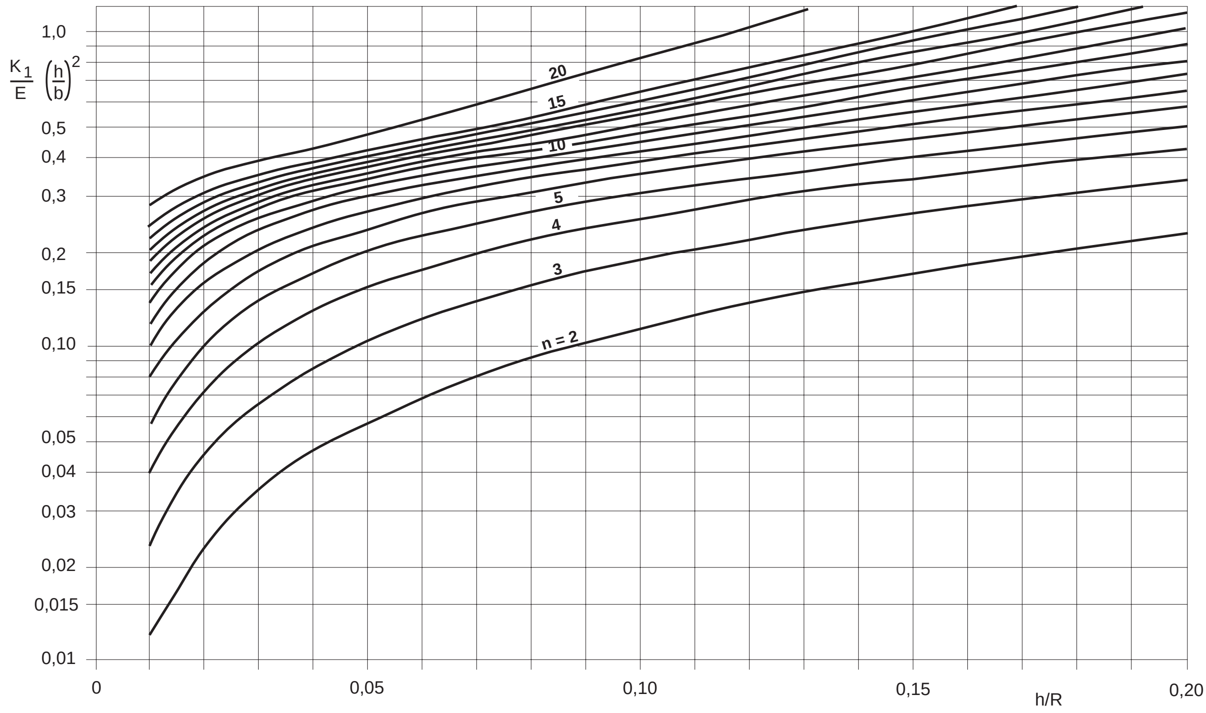

Fig. 3 Stresses k_1 for the calculation of internal flat bar stiffeners

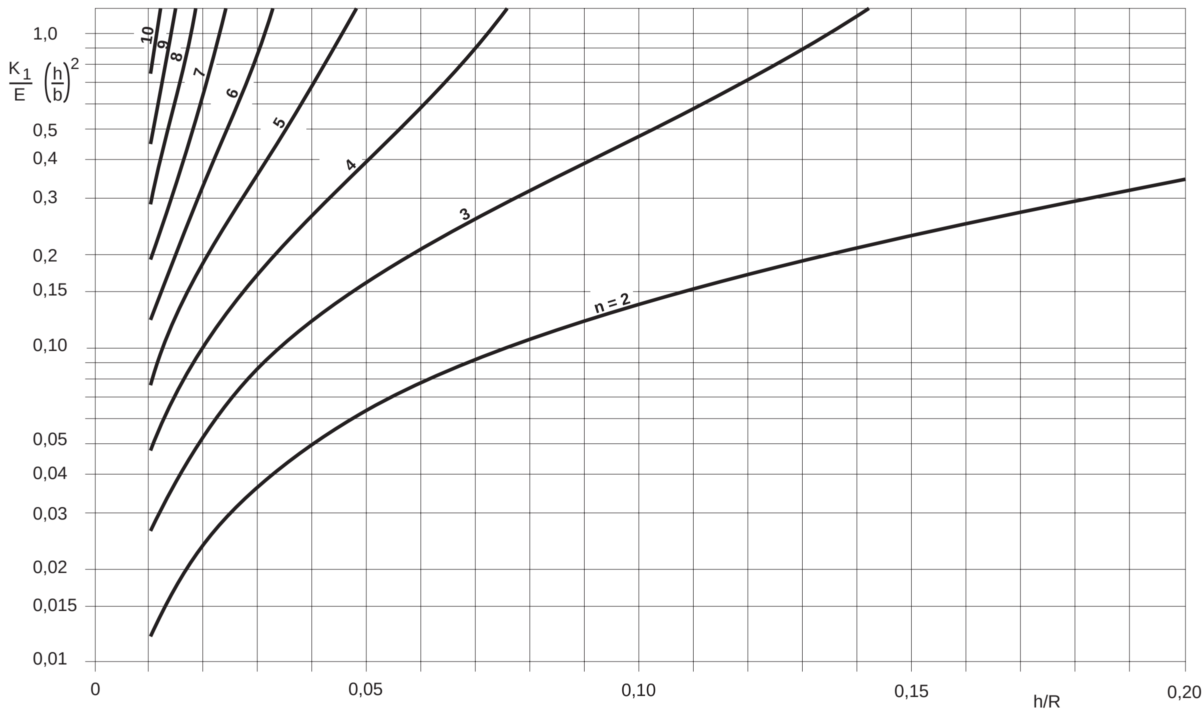

Fig. 4 Stresses k_1 for the calculation of external flat bar stiffeners

Proof of the sufficient resistance to tripping of L-, T- and I-section stiffeners can be provided by applying formula (53).

$$k \cdot S_k \leq \frac{E \cdot I_1'}{A_1 \cdot R \cdot e} \qquad (53)$$

Proof can be dispensed with if minimum seven of the following eight conditions are met:

$$e_w \geq s$$

$$e_f \geq e_w, \qquad e_f \leq 2 \cdot s$$

$$d_w \leq 20 \cdot e_w, \quad d_w \leq R/2$$

$$d_f \leq 10 \cdot e_f, \quad d_f \leq d_w/2, \quad d_f \geq d_w/4$$

D. Stiffened and Unstiffened Conical Shells

The procedure to be applied to conical shells is similar to that for cylindrical shells. Conical shells are replaced in sections by cylinders having the mean diameter and by multiplying the actual external pressure by 1/cosφ. It is assumed that the ends of the cone are fitted with "heavy" ring stiffeners. If not, a stress analysis has to be performed in accordance with F.1. Ring stiffeners are to be calculated in the manner described in C. The instructions given in B.1 are applicable to out-of-roundness values in conical shells.

E. Dished Ends and Spheres

1. General

Dished ends and spheres are to be examined for excess stresses and buckling under the loading conditions stated in A. In the case of dished ends, the stresses in the crown radius and in knuckle radius are to be investigated. Spheres are to be treated in the same way as the crown radius of dished ends.

The calculation allow for out-of-roundness of the shell up to a maximum of u = 0,04 · s/R. If larger tolerances are planned, or if the method of measurement described in H.3 results in greater out-of-roundness values, than the permissible pressure is to be checked in accordance with H.4.

2. Stresses

For the dished sections the stress is obtained by applying formula (54). For the knuckle radius the stress is obtained with formula (55), the radius R being the radius of the adjoining cylindrical jacket. The coefficient β are to be taken from reference [2] or Fig. 5. For hemispherical ends in the range of $0{,}5\sqrt{s \cdot R}$ beside the transition to the cylinder a coefficient β = 1,1 is valid.

$$\sigma = -\frac{R \cdot p}{2 \cdot s} \qquad (54)$$

$$\sigma = -\frac{p \cdot R \cdot 1{,}2 \cdot \beta}{2 \cdot s} \qquad (55)$$

3. Provision against excess stresses

The stress for the three loading conditions is obtained by applying formula (54) and (55). Sufficient safety against excess stresses is provided if the conditions (56a, b, c) are met, allowing for the absolute values of σ.

$$k \geq |\sigma| \cdot S \qquad (\text{for } p = p_N) \qquad (56a)$$

$$k \geq |\sigma| \cdot S' \qquad (\text{for } p = p_P) \qquad (56b)$$

$$k \geq |\sigma| \qquad (\text{for } p = p_Z) \qquad (56c)$$

4. Buckling

The buckling pressure p_n in the dished section for the nominal diving pressure and test diving pressure load conditions is determined by applying formula (57).

$$p_n = 0{,}366 \cdot E \cdot \left(\frac{s}{R}\right)^2 \qquad (57)$$

The buckling pressure p_n in the dished section for the collapse pressure load condition is calculated with formula (58). The elasticity moduli E_s and E_t are calculated in accordance with G. allowing for the stress determined with formula (54).

$$p_n = 0{,}84 \cdot \sqrt{E_s \cdot E_t} \cdot \left(\frac{s}{R}\right)^2 \qquad (58)$$

5. Provision against buckling

The buckling pressure for the nominal diving pressure and test diving pressure load conditions is calculated with formula (57). Sufficient safety is provided if the conditions (59a, b) are met.

The buckling pressure for the collapse pressure load condition is calculated with formula (58). Sufficient safety is provided if conditions (59c) is met.

$$p_n \geq p_N \cdot S_k \qquad (59a)$$

(for the nominal diving pressure load condition)

$$p_n \geq p_P \cdot S_k' = p_N \cdot S_1 \cdot S_k' \qquad (59b)$$

(for the test diving pressure load condition)

$$p_n \geq p_Z = p_N \cdot S_2 \qquad (59c)$$

(for the collapse pressure load condition)

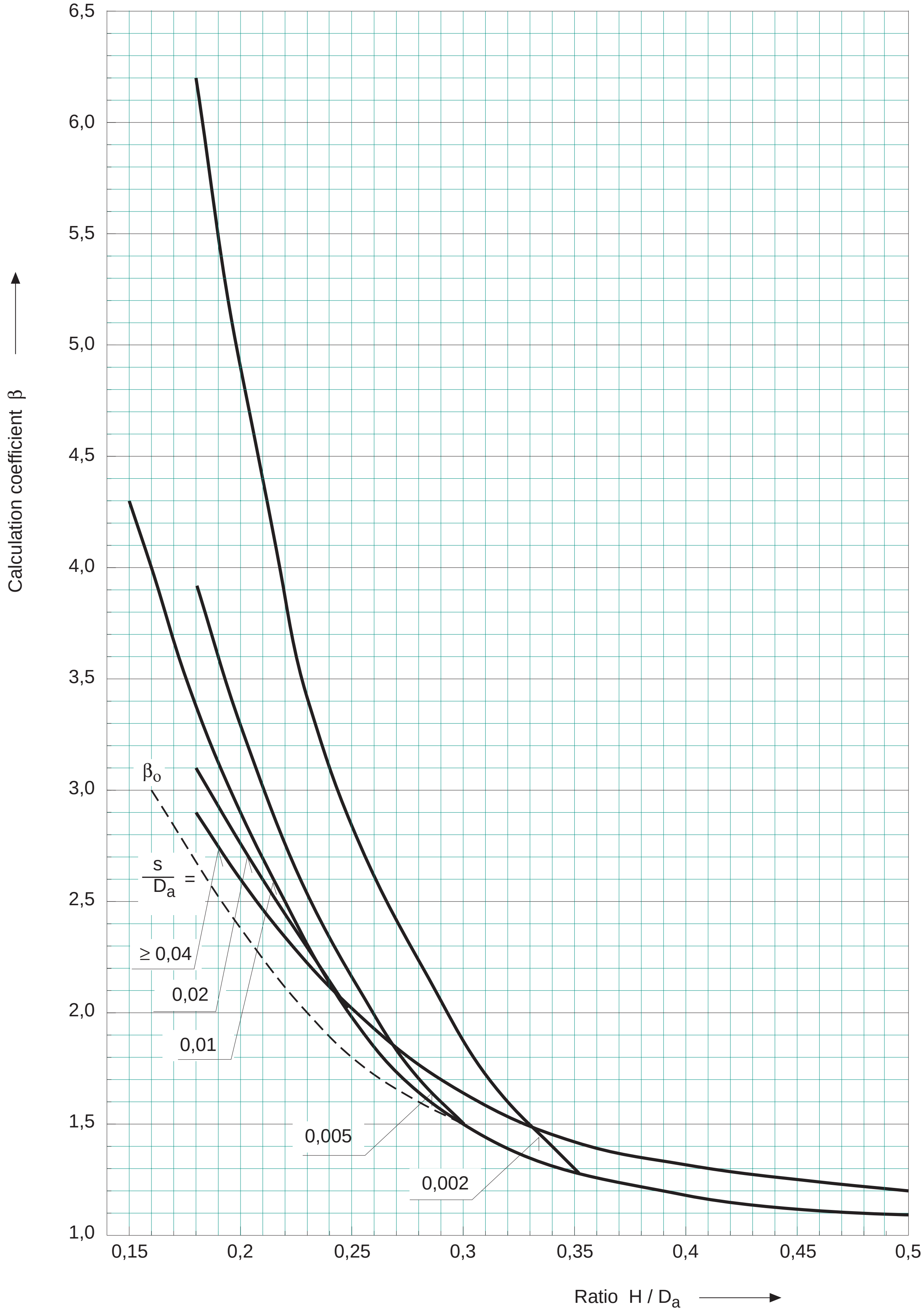

Fig. 5 Values of β for the calculation of dished ends

F. Openings and Discontinuities

1. Discontinuities

Discontinuities such as

- Connections between cylinders and conical segments
- Reinforcing rings (rings other than the ring stiffeners dealt with in C.)
- Flanges for fixing spherical shell windows

must be subjected to a stress and elongation analysis similar to that specified in [19] and [11] for the nominal diving pressure and test diving pressure load conditions. The comparison stress is determined by applying formula (1). Sufficient safety is provided if the conditions (16a, b) are met. In case of an interruption of stiffeners an adequate reinforcing has to be provided.

2. Cylinder/cylinder penetrations

Cutouts in cylinders are to be made in accordance with the Rules Part 1 – Seagoing Ships, Chapter 2 – Machinery Installations, Section 7a, D.2.3.4 and using as internal pressure a design pressure p_c calculated by applying formulae (60) and (61) - minimum with the relevant pressure of the load case. Reinforcements are to be provided as integral reinforcements.

$$p_c = \frac{2 \cdot p_N{}^2 \cdot R \cdot S}{k \cdot F \cdot s_A} \qquad (60a)$$

$$p_c = \frac{2 \cdot p_P{}^2 \cdot R \cdot S'}{k \cdot F \cdot s_A} \qquad (60b)$$

$$p_c = \frac{2 \cdot p_Z{}^2 \cdot R}{k \cdot F \cdot s_A} \qquad (60c)$$

$$F = 1 + 3 \cdot u \cdot \left(1 - \frac{0,4 \cdot R}{L_1}\right) \cdot \frac{R}{s_A}$$

$$\text{for } \frac{L_1}{R} \geq 0,4 \qquad (61a)$$

$$F = 1 \quad \text{for } \frac{L_1}{R} < 0,4 \qquad (61b)$$

3. Sphere/cylinder penetrations

Cutouts in spheres are to be made in accordance with GL Rules for the Classification and Construction, Part 1 – Seagoing Ships, Chapter 2 – Machinery Installations, Section 7a, D.4.3.3 and using as internal pressure an increased design pressure p_c calculated by applying formula (62).

$$p_c = 1,2 \cdot p_N \qquad (62a)$$

$$p_c = 1,2 \cdot p_P \qquad (62b)$$

$$p_c = 1,2 \cdot p_Z \qquad (62c)$$

G. Elasticity Moduli

The elasticity modulus for calculations in the elastic region up to the limit of proportionality is to be taken from the standard specifications for the materials concerned. For design temperatures up to 50 °C, a value of E = 206.000 N/mm² can generally be accepted for ferritic steels. For steel, a Poisson's ratio of ν = 0,3 is to be used.

In the elastic-plastic range, the elasticity moduli Es_c and E_t for steel between the limit of proportionality σ_e and the yield point k according to the stress-strain curve σ = f (ε, k, E) are to be determined by applying formulae (63) - (66).

$$z = \frac{\sigma_e}{k}$$

$$\sigma = k \cdot \left[z + (1-z) \cdot \text{tgh}\left(\frac{E \cdot \varepsilon}{(1-z) \cdot k} - \frac{z}{(1-z)} \right) \right] \qquad (63)$$

$$\varepsilon = \frac{k}{E} \cdot \left[z + (1-z) \cdot \text{artgh}\left(\frac{\sigma}{(1-z) \cdot k} - \frac{z}{(1-z)} \right) \right] \qquad (64a)$$

$$\varepsilon_{min} = z \cdot \frac{k}{E} \qquad (64b)$$

$$\varepsilon_{max} = \text{Min}\left[\text{max.remaining elongation} + \frac{k}{E}, \; \varepsilon = f(\sigma \rightarrow k) \right] \qquad (64c)$$

$$E_s = \frac{k}{\varepsilon} \cdot \left[z + (1-z) \cdot \text{tgh}\left(\frac{E \cdot \varepsilon}{(1-z) \cdot k} - \frac{z}{(1-z)} \right) \right] \qquad (65)$$

$$E_1 = E \cdot \left[1 - \text{tgh}^2\left(\frac{E \cdot \varepsilon}{(1-z) \cdot k} - \frac{z}{(1-z)} \right) \right] \qquad (66)$$

For calculations in the elastic-plastic range which were originally developed for the elastic range, the term E is to be replaced by the term E' from formula (67).

$$E' = \sqrt{E_s \cdot E_t} \qquad (67)$$

With GL's agreement, the stress-strain curve actually measured may b e used to determine the elasticity moduli in the elastic-plastic range.

In the elastic-plastic range, the Poisson's ratio is to be calculated using formula (68).

$$\nu' = \frac{1}{2} - \left(\frac{1}{2} - \nu \right) \cdot \frac{E_s}{E} \qquad (68)$$

H. Out-of-Roundness of Cylinders and Spheres

Cylindrical shells and dished ends subjected to external pressure are to be checked for out-of-roundness. If the tolerances are exceeded, the permissible external pressure is to be reduced to the value p'.

1. Measuring the out-of-roundness of cylindrical shells

The number of planes used for measuring the out-of-roundness of cylindrical pressure vessels is to be agreed with GL. For each plane, the number of measuring prints (J) shall be at least 24, and these shall be evenly distributed round the circumference. The height of arc x (j) is measured with a bridge extending over a string length $y = 4 \cdot \pi \cdot (R + s/w)/J$ (cf. Fig. 6). Form the values x (j) and the influence coefficients C, the out-of-roundness values can be calculated by applying formula (69). Table 1 gives the influence coefficients #C where J = 24. If the out-of-roundness U (j) at any measuring point exceeds a value of $U = 0{,}005 \cdot R$, then a reduced permissible pressure p' is to be determined in accordance with 2.

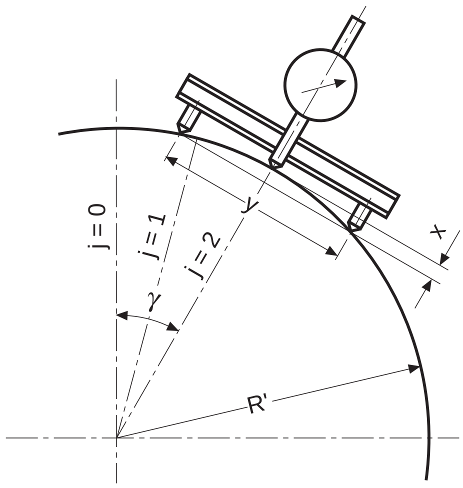

Fig. 6 Measuring the out-of-roundness of a cylindrical shell

$$U_j = \sum_{i=0}^{J-1} x_i \cdot C_{|i-j|} \qquad (69)$$

Example of the out-of-roundness U at measuring point j = 2 where J = 24:

$$U_2 = x_0 \cdot C_2 + x_1 \cdot C_1 + x_2 \cdot C_0 + x_3 \cdot C_1 + ...$$

$$+ x_{21} \cdot C_{19} + x_{22} \cdot C_{20} + x_{23} \cdot C_{21}$$

Table 1 Influence factors C_i where J = 24

i - j	$C_{\|i-j\|}$	i - j	$C_{\|i-j\|}$
0	1,76100	12	0,60124
1	0,85587	13	0,54051
2	0,12834	14	0,36793
3	– 0,38800	15	0,11136
4	– 0,68359	16	– 0,18614
5	– 0,77160	17	– 0,47097
6	– 0,68487	18	– 0,68487
7	– 0,47097	19	– 0,77160
8	– 0,18614	20	– 0,68359
9	0,11136	21	– 0,38800
10	0,36793	22	0,12834
11	0,54051	23	0,85587

2. Calculation of permissible pressure for cylindrical shell with an out-of-roundness u > 0,005

The bending stress is determined for al measuring points by the choice of a reduced permissible pressure p' and by applying formula (70). The total stress is found with formula (74) and the reduced permissible pressure p' with formula /75) by a process of iteration, the n-related value for formula (17) being substituted for the pressure p_n. The mean radius R' is to be determined by measuring the circumference.

$$\sigma_b = \frac{E \cdot s}{2R^2(1-\nu^2)} \sum_{n=2}^{J/2} \left[\left(n^2 - 1\right) + \nu \left(\frac{\pi \cdot R}{L_1}\right)^2 \right]$$

$$\cdot \left[\frac{p'}{p_n - p'}\right] \cdot \left[a_n \cdot \sin(n \cdot \gamma) + b_n \cdot \cos(n \cdot \gamma)\right] \qquad (70)$$

$$\gamma = \frac{2 \cdot \pi}{J} \cdot i \qquad (71)$$

$$a_n = \frac{2}{J} \cdot \sum_{i=0}^{J-1} \left(R' + U_i\right) \cdot \sin(n \cdot \gamma) \qquad (72)$$

$$b_n = \frac{2}{J} \cdot \sum_{i=0}^{J-1} \left(R' + U_i\right) \cdot \cos(n \cdot \gamma)$$

for $n \neq J/2$ (73a)

$$b_n = \frac{1}{J} \cdot \sum_{i=0}^{J-1} \left(R' + U_i\right) \cdot \cos(n \cdot \gamma)$$

for $n = J/2$ (73b)

$$k \geq \frac{p' \cdot R}{s} + \sigma_b \qquad (74)$$

$$p' \geq \frac{p'}{S} + \left(p - \frac{p'}{S}\right) \cdot \frac{0{,}005 \cdot R}{U_{max}} \qquad (75)$$

3. Measuring the out-of-roundness of spheres

The height of arc x' is measured with a bridge gauge (cf. Fig. 7), the string length y being calculated with formulae (76) and (79). The out-or-roundness U is determined with formula (78). If the out-of-roundness is greater than u = 0,04 · s/R, a reduced permissible pressure p' is to be determined in accordance with 4.

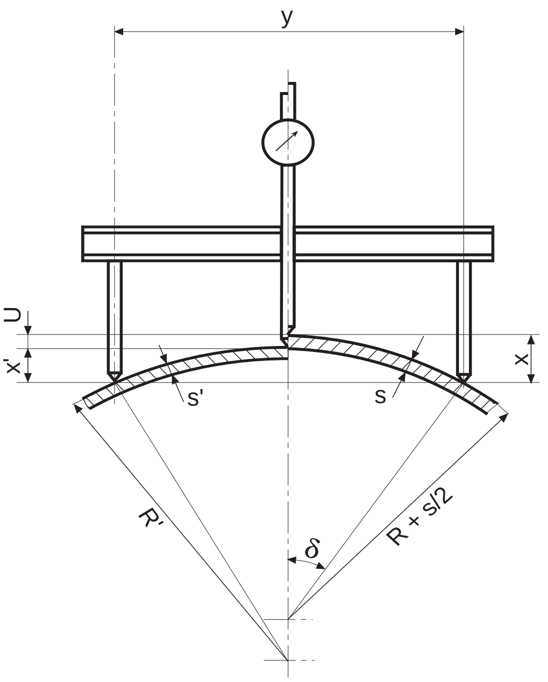

Fig. 7 Measuring the out-of-roundness of a sphere

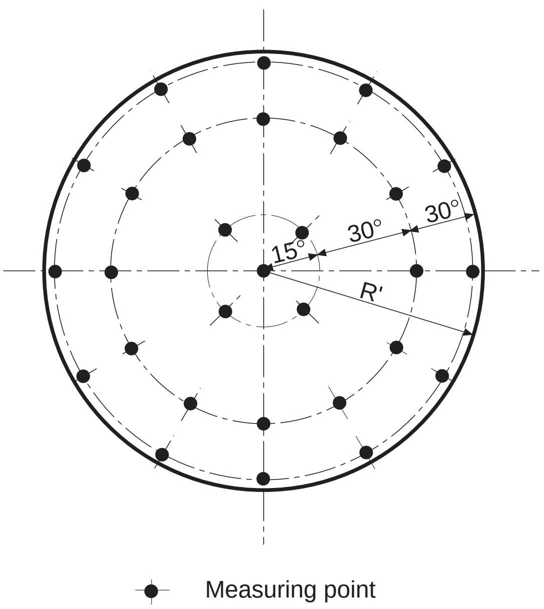

Fig. 8 Distribution of measuring points over hemisphere

$$y = 2 \cdot \left(R + \frac{s}{2}\right) \cdot \sin\delta \tag{76}$$

$$x = \left(R + \frac{s}{2}\right) \cdot \left(1 - \cos\delta\right) \tag{77}$$

$$U = x - x' = u \cdot R \tag{78}$$

$$\delta = \frac{1{,}1}{\left(1 - \nu^2\right)} \cdot \sqrt{\frac{s}{R + \frac{s}{2}}} \tag{79}$$

the distribution of the measuring points is shown in Fig. 8. Two measurements are to be made at each point: one in the plane of the central axis, the other at right angles to it.

4. Calculation of permissible pressure for spheres with an out-of-roundness u > 0,04 · s/R

The reduced permissible pressure p' is calculated with formula (80) allowing for the actual radius of curvature R' and the minimum wall thickness s' occurring in the measuring range y (taking account of any reductions for wear and corrosion). The radius of curvature R' is determined with formula (81).

$$p' = p \cdot \left(\frac{R + \frac{s}{2}}{R'}\right)^2 \cdot \left(\frac{s'}{2}\right)^2 \le p \tag{80}$$

$$R' = \frac{x'}{2} + \frac{y^2}{8 \cdot x'} \tag{81}$$

I. Tolerances of Ring Stiffeners

A check is to be carried out on ring stiffeners to determine whether the following tolerances have been maintained:

– Girth width (on T-section flanges, the whole width): - 0/+ 5 mm

– Girth and web thickness: - 0/+ t

  The tolerance t depends on the conditions of supply for the material. (If the material supply specification allows negative tolerances, these are to be allowed for in the calculations)

– Height of ring (in the case of built-up profiles the height of the entire ring): - 2 %/+ 5 % of the total height

– Uneveness of web and girth (measured over height of web and girth respectively): 0/1 % of web and girth height respectively

– Symmetry of flange in relation to web (applicable to I- and T-section stiffeners: the difference from the edge of the girth to the web on both sides of the web): 0/4 mm difference

– L_1 distances (distances between "light" stiffeners and separating "light" from "heavy" stiffeners): - 5,0/+ 5,0 mm

– L_2 distances (distances between "heavy" stiffeners or ends or separating "heavy" stiffeners from ends): - 15,0/+ 5,0 mm

– Angularity of web in relation to wall or main axis: - 2°/+ 2°

– Angularity of flange in relation to web: - 3°/+ 3°

All dimensional deviations are to be measured eight times on each stiffener at points equally spaces round the circumference. If the aforementioned tolerances are exceeded, corrective machining and/or manual work is to be carried out on the stiffener and/or the calculation is to be repeated with corrected dimensions.

J. Program Sequences for Iterative Calculations

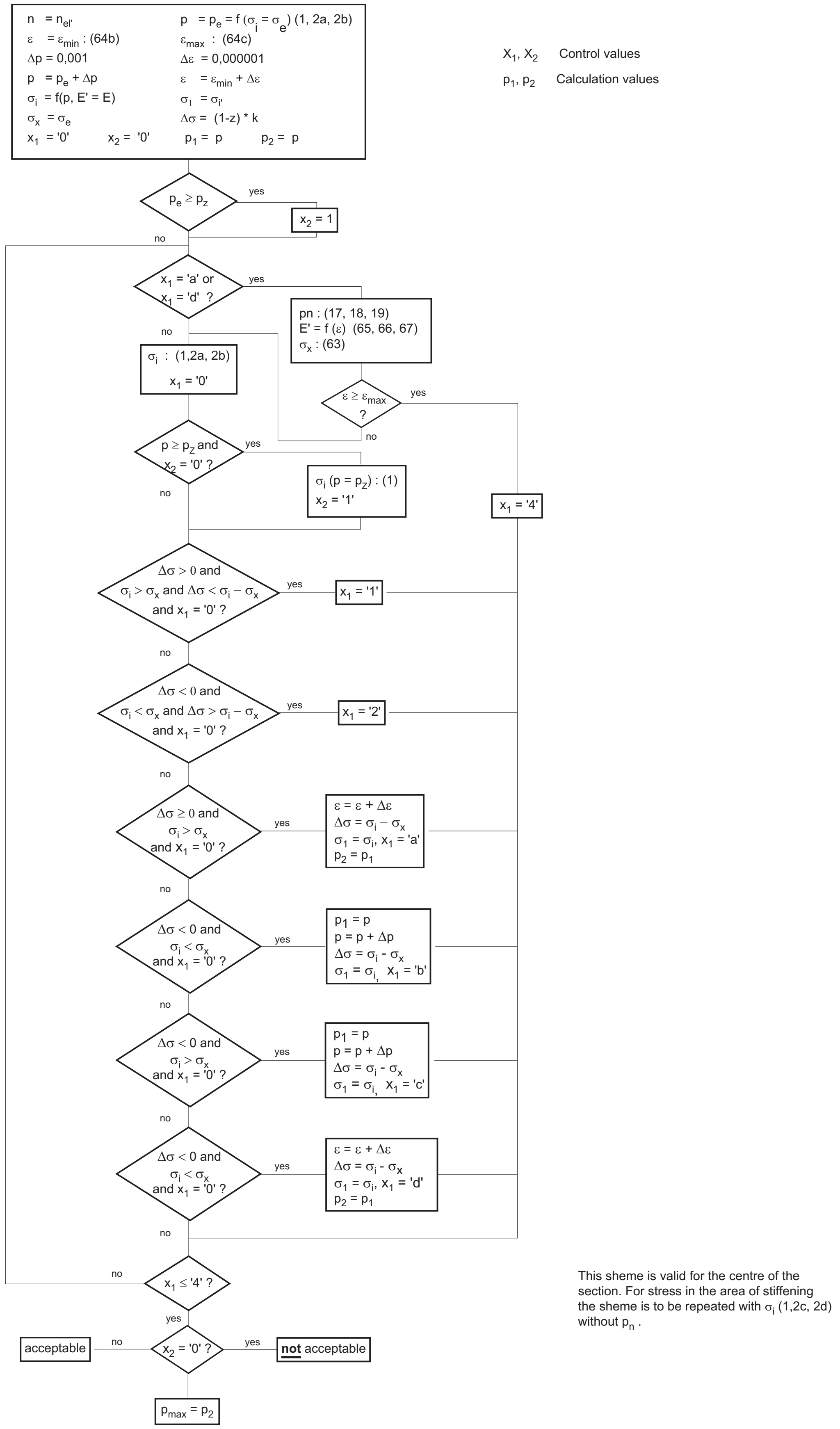

Fig. 9 Cylindrical shells, stresses and asymmetric buckling in the elastic-plastic range

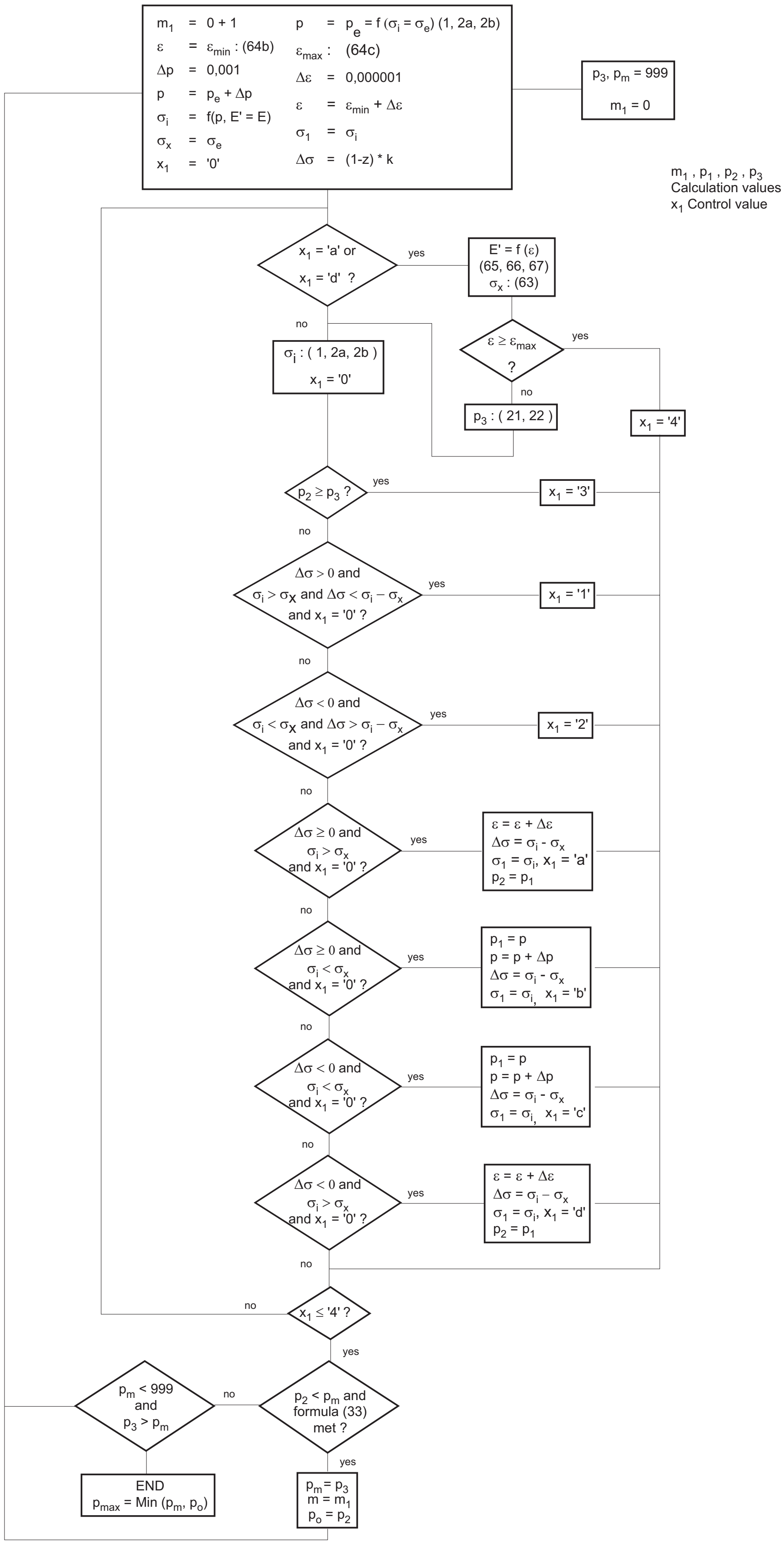

Fig. 10 Cylindrical shells, symmetric buckling in the elastic-plastic range

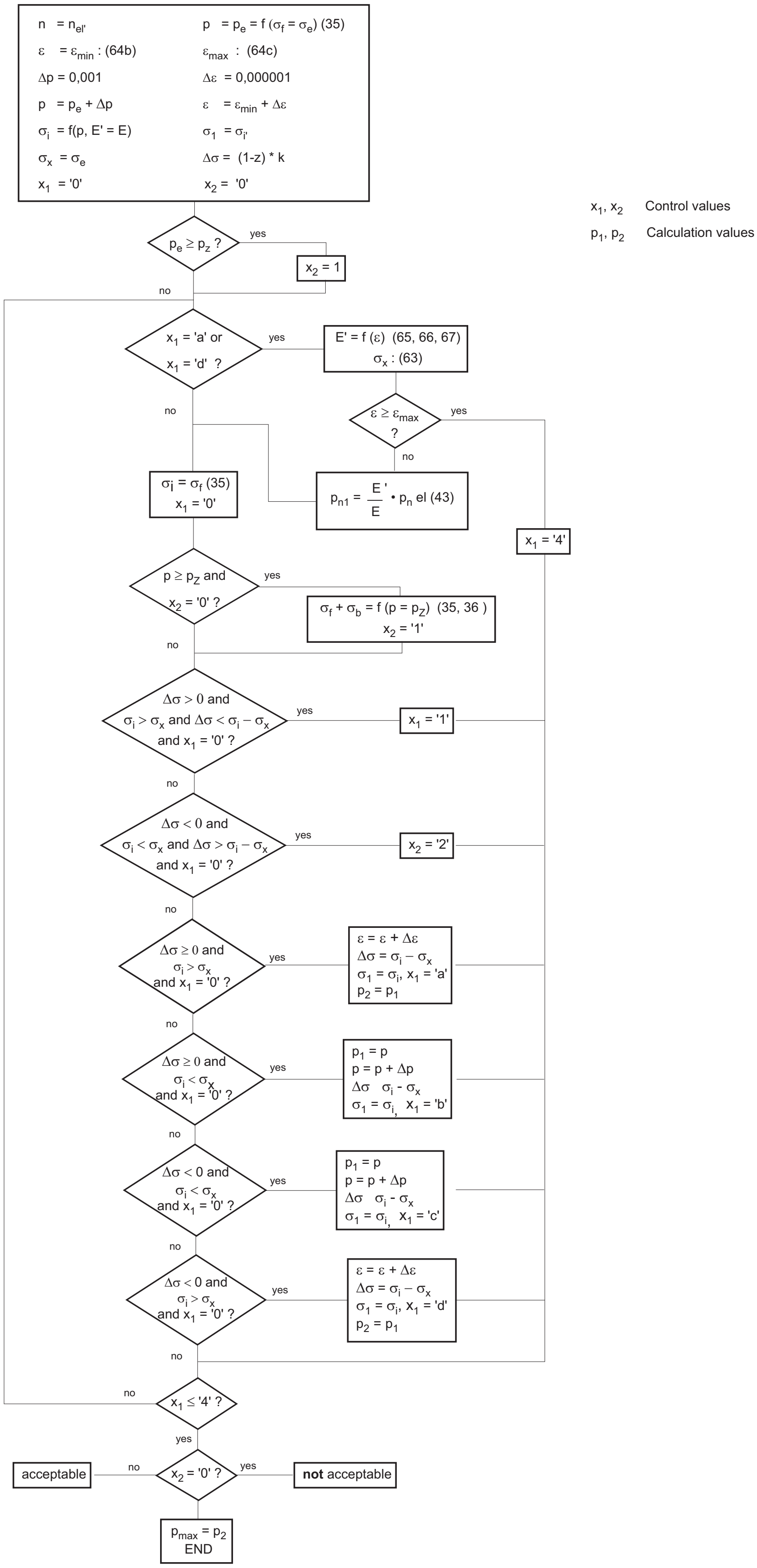

Fig. 11 "Light" stiffeners in the elastic-plastic range

K. Symbols and Units

A	[mm²]	modified area of stiffener ring
A_1	[mm²]	cross-sectional area of stiffener ring
b	[mm]	width of stiffener ring in contact with shell
C	[1]	influence coefficient for calculating out-of-roundness
$C_0 ... C_3$	[1]	calculation factors for symmetric buckling
C_4	[1]	calculation factor for asymmetric buckling, "heavy" stiffener rings
$C_5 ... C_{11}$	[1]	calculation factor for stress in cylindrical shell
d_f	[mm]	width of girth of L-, T- or I-stiffener ring from web to edge of flange
d_w	[mm]	height of web L-, T- or I-stiffener ring
e_f	[mm]	girth thickness of L-, T- or I-stiffener
e_w	[mm]	web thickness of L-, T- or I-stiffener
e	[mm]	distance from stiffener ring centroid to centre of cylindrical wall
e_1	[mm]	distance from stiffener ring centroid to centre of cylinder wall including effective length of shell L_e
e_2	[mm]	distance between stiffener ring centroid, including effective length of shell L_e, and flange face facing away from cylinder wall
E	[N/mm²]	modulus of elasticity
E'	[N/mm²]	modulus of elasticity elastic-plastic range, see formula (67)
E_s	[N/mm²]	secant modulus, see formula (65)
E_t	[N/mm²]	tangent modulus, see formula (66)
F	[1]	factor for calculations in plastic range, see formula (61)
$F_1 ... F_4$	[1]	geometric factors, see formulae (3a) to (3d)
G	[1]	pressure ratio, see formula (13)
h	[mm]	height of stiffener ring
H	[mm]	depth of dished end
$H_1 ... H_4$	[1]	calculation factors for symmetric buckling
I_1	[mm⁴]	second moment of area of stiffener ring cross-section about axis through centroid parallel to cylinder axis
I_1'	[mm⁴]	second moment of area of stiffener ring cross-section about axis through centroid vertical to cylinder axis
I_e	[mm⁴]	second moment of area of stiffener ring cross-section, including effective length of shell acting with stiffener ring, about axis through centroid parallel to cylinder axis
J	[1]	number of measuring points for checking out-of-roundness
j	[1]	number (0 ... J-1) of measuring point used to check out-of-roundness
k	[N/mm²]	yield strength R_{eH20}
k_1	[N/mm²]	stress in flat bar, cf. formula (51) and Abb. 3 and 4
K_o	[1]	stress ratio, cf. formula (15)
K_1	[1]	stress coefficient, cf. formula (31)
L	[mm]	spacing between effective stiffeners
L_1	[mm]	spacing between two "light" stiffeners
L_2	[mm]	spacing between two "heavy" stiffeners
L_3	[mm]	total length or length between bulkheads
L_e	[mm]	effective length of shell
m	[1]	mode of collapse, symmetric buckling
n	[1]	mode of collapse, asymmetric buckling
N	[1]	coefficient, cf. formula (37)
p	[N/mm²]	external design pressure
p'	[N/mm²]	external pressure reduced owing to out-of-roundness
p_o	[N/mm²]	pressure, membrane stress part
p_c	[N/mm²]	design pressure for calculation of opening
p_e	[N/mm²]	pressure in which the stress reaches the elastic-plastic range
p_g	[N/mm²]	total buckling pressure
p_k	[N/mm²]	tripping pressure

p_m	[N/mm^2]	buckling pressure, symmetric buckling
p_n	[N/mm^2]	buckling pressure, asymmetric buckling
p_{n1}	[N/mm^2]	buckling pressure, asymmetric buckling, "light" stiffener
p_{n2}	[N/mm^2]	buckling pressure, asymmetric buckling, "heavy" stiffener
p_N	[N/mm^2]	nominal diving pressure (1st load condition)
p_P	[N/mm^2]	test diving pressure (2nd load condition)
p_Z	[N/mm^2]	collapse pressure (3rd load condition)
p*	[N/mm^2]	critical pressure, cf. formula (12)
p**	[N/mm^2]	critical pressure, elastic-plastic, cf. formula (22)
R	[mm]	mean radius of wall
R_o	[mm]	radius of stiffener ring centroid including effective length L_e
R_1	[mm]	radius of standing flange of stiffener ring
R'	[mm]	measured outside radius
s	[mm]	thickness of shell/sphere without abrasion and corrosion
s'	[mm]	measured wall thickness without abrasion and corrosion
s_A	[mm]	thickness of shell/sphere adjoining opening
S	[1]	safety factor applied to yield strength R_{eH20} at nominal pressure
S'	[1]	safety factor applied to yield strength R_{eH20} at test diving pressure
S_k	[1]	safety factor against instability at nominal pressure
S'_k	[1]	safety factor against instability at test diving pressure
t	[mm]	tolerance
U	[mm]	out-of-roundness: $U = u \cdot R$
u	[1]	out-of-roundness in relation to R
x	[mm]	theoretic height of arc
x'	[mm]	effective height of arc
y	[mm]	string dimension of measurement device
z	[1]	proportional ratio (= σ_e/k)
α	$\left[\frac{1}{mm}\right]$	shape factor, cf. formula (14)
α_1	$\left[\frac{1}{mm}\right]$	shape factor, cf. formula (32)
β	[1]	coefficient for dished ends, cf. reference [2] or Fig. 5
β_{n1}	[1]	coefficient, cf. formula (18)
β_{n2}	[1]	coefficient, cf. formula (40)
γ	[rad]	angle used in increasing the out-of-roundness of cylinders
δ	[rad]	angle used in measuring out-of-roundness of spheres
λ	[1]	coefficient, cf. formulae (19), (41)
ν	[1]	Poisson's ratio, elastic
ν'	[1]	Poisson's elastic-plastic, cf. formula (68)
φ	[rad]	angle of cone (between wall and axis)
σ_b	[N/mm^2]	bending stress in case of out-of-roundness
σ_e	[N/mm^2]	proportional limit
σ_f	[N/mm^2]	compression stress in girth
σ_{fb}	[N/mm^2]	bending stress in girth
σ_i	[N/mm^2]	stress intensity
σ_o	[N/mm^2]	stress (calculate value)
σ_x	[N/mm^2]	stress in longitudinal direction
σ_φ	[N/mm^2]	stress in circumferential direction
σ_{zul}	[N/mm^2]	permissible stress

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